Joint Tx-Rx Beamforming Design for Multicarrier MIMO Channels: A Unified Framework for Convex Optimization

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Abstract

This paper addresses the joint design of transmit and receive beamforming or linear processing (commonly termed linear precoding at the transmitter and equalization at the receiver) for multicarrier multiple-input multiple-output (MIMO) channels under a variety of design criteria. Instead of considering each design criterion in a separate way, we generalize the existing results by developing a unified framework based on considering two families of objective functions that embrace most reasonable criteria to design a communication system: Schur-concave and Schur-convex functions. Once the optimal structure of the transmit-receive processing is known, the design problem simplifies and can be formulated within the powerful framework of convex optimization theory, in which a great number of interesting design criteria can be easily accommodated and efficiently solved even though closed-form expressions may not exist. From this perspective, we analyze a variety of design criteria and, in particular, we derive optimal beamvectors in the sense of having minimum average bit error rate (BER). Additional constraints on the Peak-to-Average Ratio (PAR) or on the signal dynamic range are easily included in the design. We propose two multi-level water-filling practical solutions that perform very close to the optimal in terms of average BER with a low implementation complexity. If cooperation among the processing operating at different carriers is allowed, the performance improves significantly. Interestingly, with carrier cooperation, it turns out that the exact optimal solution in terms of average BER can be obtained in closed-form.

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1 Introduction

Multiple-input multiple-output (MIMO) channels arise in many different scenarios such as when a bundle of twisted pairs in digital subscriber lines (DSL) is treated as a whole [1], when multiple antennas are used at both sides of a wireless link [2], or simply when a frequency-selective channel is properly modeled by using, for example, transmit and receive filterbanks [3]. In particular, MIMO channels arising from the use of multiple antennas at both the transmitter and at the receiver have recently attracted a significant interest because they provide an important increase in capacity over single-input single-output (SISO) channels under some uncorrelation conditions [4, 5].

In terms of spectral efficiency, a MIMO system should be designed to approach the capacity of the channel [6, 2, 7]. In light of this observation, a frequency-selective MIMO channel can be dealt with by taking a multicarrier approach which is a well-known capacity-lossless structure and allows to treat each carrier as a flat MIMO channel [2, 8]. A capacity-achieving design dictates that the channel matrix at each carrier must be diagonalized and then a *water-filling* power allocation must be used on the spatial subchannels (or channel eigenmodes) of all carriers [6, 2, 7]. Note that this requires channel state information (CSI) available at both ends of the link, which we assume in the rest of the paper. In theory, this solution has the implication that an ideal Gaussian code should be used on each spatial eigenmode and carrier according to its allocated power [6]. In practice, however, each Gaussian code is substituted by a simple (and suboptimal) signal constellation and a practical (and suboptimal) coding scheme (if any). The complexity of such a solution is still significative since each channel eigenmode requires a different combination of signal constellation and code depending on the allocated power. To reduce the complexity, the system can be constrained to use the same constellation and code in all channel eigenmodes (possibly optimizing the utilized bandwidth to transmit only over those eigenmodes with a sufficiently high gain), *i.e.*, an equal-rate transmission. Examples of this pragmatic and simple solution are found in the European standard HIPERLAN/2 [9] and in the US standard IEEE 802.11 [10] for Wireless Local Area Networks (WLAN).

Assuming that the specific signal constellations and coding schemes for all the substreams have been selected either after some bit distribution method or simply by taking a simple uniform bit distribution, it is then possible to further optimize the system to improve the quality of each of the communication links. In particular, we consider the joint design of linear processing at both ends of the link (commonly referred to as linear precoder at the transmitter and equalizer at the receiver) according to a variety of criteria as we now review. In [11, 12, 13, 3], the sum of the mean square error (MSE) of all channel substreams (the trace of the MSE matrix) was used as the objective to minimize under an average power constraint. This criterion was generalized by using a weighted sum (weighted trace) in [14]. In [3], a maximum signal to interference-plus-noise ratio (SINR) criterion with a zero-forcing (ZF) constraint was also considered. For these criteria, the original complicated design problem is greatly simplified because the channel turns out to be diagonalized by the transmit-receive processing. In [15], the determinant of the MSE matrix was minimized and the diagonal structure was found to be optimal as well. In [16], the results were extended to the case of a peak power constraint (maximum eigenvalue constraint) with similar results.

We remark that the channel-diagonalizing property is of paramount importance in order to be able to solve the problem. The main interest of the diagonalizing structure is that it allows a *scalarization* of the problem (meaning that all matrix equations are substituted with scalar ones) with the consequent great simplification. In light of the optimality of the channel-diagonalizing structure in all the aforementioned examples (including the capacity-achieving solution), one may wonder whether the same holds for other criteria. Examples of other reasonable criteria to design a communication system are the minimization of the maximum bit error rate (BER) of the substreams, the minimization of the average BER, or the maximization of the minimum SINR of the substreams. In these cases, it is not clear whether one can assume a diagonal structure as was obtained in the previous cases.

In this paper, we consider different design criteria based on optimizing the MSE's, the SINR's, and also the BER's directly. Instead of considering each design criterion in a separate way, we develop a unifying framework and generalize the existing results by considering two families of objective functions that embrace most reasonable criteria to design a communication system: Schur-concave and Schur-convex functions which arise in majorization theory [17]. For Schur-concave objective functions, the channel-diagonalizing structure is always optimal, whereas for Schur-convex functions, an optimal solution diagonalizes the channel only after a very specific rotation of the transmitted symbols. Once the optimal structure of the transmit-receive processing is known, the design problem simplifies and can be formulated within the powerful framework of convex optimization theory, in which a great number of interesting design criteria can be easily accommodated and efficiently solved even though closed-form expressions may not exist. We analyze a variety of criteria and, in particular, we derive optimal beamvectors in the sense of having minimum average BER. A convex optimization approach for the simple case of utilizing a single spatial eigenmode (in other words, using a single beamforming per carrier) was also taken in [18]. Additional constraints on the Peak-to-Average Ratio (PAR) or on the signal dynamic range are easily included in the design within the convex optimization framework. We propose two multi-level water-filling practical solutions that perform very close to the optimal in terms of average BER with a low implementation complexity. If cooperation among the processing operating at different carriers is allowed, the performance improves significantly. Interestingly, with carrier cooperation, it turns out that the optimal solution in the sense of minimum average BER can be obtained in closed-form.

The paper is structured as follows. In Section 2, a brief preliminary description of convex optimization problems and of majorization theory is given. The signal model is introduced in Section 3. The main result of the paper (the optimal structure for Schur-concave and Schur-convex objective functions) is given in Section 4. Section 5 is devoted to the systematic design of beamforming under the framework of convex optimization theory. In Section 6, additional constrains such as to control the PAR are considered. Simulation results are given in Section 7. The final conclusions of the paper are summarized in Section 8.

The following notation is used. Boldface upper-case letters denote matrices, boldface lower-case letters denote column vectors, and italics denote scalars. The super-scripts $(\cdot)^T$, $(\cdot)^*$, and $(\cdot)^H$ denote transpose, complex conjugate, and Hermitian operations respectively. $[\mathbf{X}]_{i,j}$ (also $[\mathbf{X}]_{ij}$) and $[\mathbf{X}]_{:,j}$ denote the (ith, jth) element and jth column of matrix $\mathbf{X}$ respectively. By $\mathbf{A} \geq \mathbf{B}$ we mean that $\mathbf{A} - \mathbf{B}$ is positive semidefinite. The trace, determinant, and Frobenius norm of a matrix are denoted by $\mathrm{Tr}(\cdot)$, $|\cdot|$, and $\|\cdot\|_F$ respectively. By $\mathrm{diag}(\{\mathbf{X}_k\})$ we denote a block-diagonal matrix with diagonal blocks given by the set

$\{\mathbf{X}_k\}$. The gradient of a function with respect to $\mathbf{x}$ is written as $\nabla_{\mathbf{x}} f(\mathbf{x})$. We define $(x)^+ \triangleq \max(0, x)$.

2 Preliminaries

In Section 5, a variety of objective functions are considered under the powerful framework of convex optimization theory [19, 20, 21]. For this purpose, we first give an overview in subsection 2.1 of the potential and advantages of this framework. Roughly speaking, one can say that once a problem has been expressed in convex form, it has been solved. However, before being able to express the different criteria in convex form, a simplification of the problem is necessary. Majorization theory [17] provides us with useful tools to simplify many matrix-valued problems which we review in subsection 2.2.

2.1 Convex Optimization Problems

A general *convex optimization problem* (*convex program*) is of the form [19, 21]:

$$\begin{array}{lll} \min\limits_{\mathbf{x}} & f_0(\mathbf{x}) & \\ \text{s.t.} & f_i(\mathbf{x}) \leq 0 & 1 \leq i \leq m, \\ & h_i(\mathbf{x}) = 0 & 1 \leq i \leq p, \end{array}$$

where $\mathbf{x} \in \mathbb{R}^n$ is the optimization variable, $f_0(\mathbf{x}), \cdots, f_m(\mathbf{x})$ are convex functions, and $h_1(\mathbf{x}), \cdots, h_p(\mathbf{x})$ are linear functions (more exactly affine functions). The function f_0 is the *objective function* or *cost function*. The inequalities $f_i(\mathbf{x}) \leq 0$ are called *inequality constraints* and the equations $h_i(\mathbf{x}) = 0$ are called *equality constraints*. When the functions f_i and h_i are linear (affine), the problem is called a *linear program* (LP) and is much simpler to solve.

Many analysis and design problems arising in engineering can be cast (or recast) in the form of a convex optimization problem. In general, some manipulations are required to convert the problem into a convex one (unfortunately, this is not always possible). The interest of expressing a problem in convex form is that, although an analytical solution may not exist and the problem may be difficult to solve (it may have hundreds of variables and a nonlinear nondifferentiable objective function), it can still be solved (numerically) very efficiently, both in theory and practice [21]. Another interesting feature of expressing a problem in convex form is that additional constraints can be straightforwardly added as long as they are convex. For example, in the problem addressed in this paper, it is very simple to add constraints to control the dynamic range of the power amplifier [22] and the PAR of the transmitted signal (c.f. Section 6).

Convex programming has been used in related areas such as FIR filter design [23], antenna array pattern synthesis [24], power control for interference-limited wireless networks [25], and transmit downlink beamforming in a multiuser scenario with a multi-antenna base station [22].

Solving Convex Optimization Problems

In some cases, convex optimization problems can be analytically solved using the Karush-Kuhn-Tucker (KKT) optimality conditions and closed-form expressions can be obtained. In general, however, one must resort to iterative methods [19, 21]. In the last ten years, there has been considerable progress and development of efficient algorithms for solving wide classes of convex optimization problems. Recently developed *interior-point methods* can be used to iteratively solve convex problems efficiently in practice by dealing with the constrained problem as a sequence of unconstrained problems in which a Newton method can be efficiently used. This was an important breakthrough achieved by Nesterov and Nemirovsky in 1988. They showed that interior-point methods (initially proposed only for linear programming by Karmarkar in 1984) can, in principle, be generalized to all convex optimization problems. In [26] a very general framework was developed for solving convex optimization problems using interior-point methods. In addition, the difference between the objective value at each iteration and the optimum value can be upper-bounded using duality theory [19, 21]. This allows the utilization of nonheuristic stopping criteria such as stopping when some prespecified resolution has been reached. Another interesting family of iterative methods are *cutting-plane methods* [21].

2.2 Majorization Theory

We introduce the basic notion of majorization and state some important results (see [17] for a complete reference of the subject). Majorization makes precise the vague notion that the components of a vector $\mathbf{x}$ are "less spread out" or "more nearly equal" than the components of a vector $\mathbf{y}$.

Definition 1 *For any* $\mathbf{x} \in \mathbb{R}^n$*, let*

$$x_{[1]} \geq \cdots \geq x_{[n]}$$

denote the components of $\mathbf{x}$ *in decreasing order (also termed order statistics of* $\mathbf{x}$*)*

Definition 2 *[17, 1.A.1] Let* $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$*. Vector* $\mathbf{x}$ *is majorized by vector* $\mathbf{y}$ *(or* $\mathbf{y}$ *majorizes* $\mathbf{x}$*) if*

$$\sum_{i=1}^{k} x_{[i]} \leq \sum_{i=1}^{k} y_{[i]} \qquad 1 \leq k \leq n-1$$
$$\sum_{i=1}^{n} x_{[i]} = \sum_{i=1}^{n} y_{[i]}$$

and represent it by $\mathbf{x} \prec \mathbf{y}$.

Definition 3 *[17, 3.A.1] A real-valued function* ϕ *defined on a set* $\mathcal{A} \subseteq \mathbb{R}^n$ *is said to be Schur-convex on* $\mathcal{A}$ *if*

$$\mathbf{x} \prec \mathbf{y} \quad \text{on } \mathcal{A} \quad \Rightarrow \quad \phi(\mathbf{x}) \leq \phi(\mathbf{y}).$$

Similarly, ϕ *is said to be Schur-concave on* $\mathcal{A}$ *if*

$$\mathbf{x} \prec \mathbf{y} \quad \text{on } \mathcal{A} \quad \Rightarrow \quad \phi(\mathbf{x}) \geq \phi(\mathbf{y}).$$

As a consequence, if ϕ *is Schur-convex on* $\mathcal{A}$ *then* $-\phi$ *is Schur-concave on* $\mathcal{A}$ *and vice-versa.*

It is important to remark that the sets of Schur-concave and Schur-convex functions do no form a partition of the set of all functions. In fact, neither the two sets are disjoint (the intersection is not empty) nor they cover the entire set of all functions.

Lemma 1 *[17, 9.B.1] Let* $\mathbf{R}$ *be an* $n \times n$ *Hermitian matrix with diagonal elements denoted by the vector* $\mathbf{d}$ *and eigenvalues denoted by the vector* $\boldsymbol{\lambda}$*, then*

$$\mathbf{d} \prec \boldsymbol{\lambda}.$$

Lemma 2 *[17, p. 7] Let* $\mathbf{x} \in \mathbb{R}^n$ *and* $\mathbf{1} \in \mathbb{R}^n$ *denote the constant vector with* $1_i \triangleq \sum_{j=1}^{n} x_j/n$*, then*

$$\mathbf{1} \prec \mathbf{x}.$$

Lemma 3 *[17, 9.B.2] For any* $\mathbf{x} \in \mathbb{R}^n$*, there exists a real symmetric (and therefore Hermitian) matrix with equal diagonal elements and eigenvalues given by* $\mathbf{x}$*.*

3 Signal Model

We consider a communication system with n_T transmit and n_R receive dimensions. This gives rise to a MIMO channel that can be represented by a channel matrix. Many different communication channels can be expressed under the unified notation of a channel matrix such as a frequency-selective channel employing transmit and receive filterbanks [3], a bundle of twisted pairs in DSL [1], or a wireless multi-antenna system [27, 2]. Although the results in this paper are valid for any MIMO channel, we focus on a wireless multi-antenna system to gain insight into beamforming issues traditionally associated to arrays of antennas.

To deal easily with the frequency-selectivity of the channel, we take a multicarrier approach without loss of optimality (since it is known to be a capacity-lossless structure [2, 8]):

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{s}_k + \mathbf{n}_k \qquad 1 \leq k \leq N \tag{1}$$

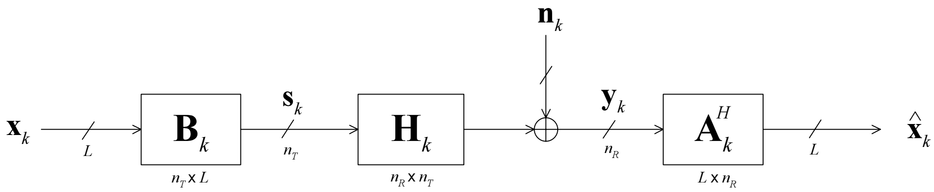
where k denotes the carrier index, N is the number of carriers, $\mathbf{s}_k \in \mathbb{C}^{n_T \times 1}$ is the transmitted vector, $\mathbf{H}_k \in \mathbb{C}^{n_R \times n_T}$ is the channel matrix, $\mathbf{y}_k \in \mathbb{C}^{n_R \times 1}$ is the received signal vector, and $\mathbf{n}_k \in \mathbb{C}^{n_R \times 1}$ is a zero-mean circularly symmetric complex Gaussian noise vector with arbitrary covariance matrix $\mathbf{R}_{n_k}$, *i.e.*, $\mathbf{n}_k \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{R}_{n_k}\right)$. The channel is assumed fixed during the transmission of a block and known at both sides of the communication link as well as the noise covariance matrix.

At each carrier k, the matrix channel has $K_k \leq \min\left(n_T, n_R\right)$ *channel eigenmodes* or *spatial sub-channels* (*i.e.*, nonvanishing singular values of the channel matrix) [2]. We can use them as a means of spatial multiplexing [28] to transmit simultaneously L_k symbols by having L_k *established substreams*. Notice that established substreams and spatial subchannels (or channel eigenmodes) are different concepts which may or may not coincide depending on whether the channel is diagonalized or not (c.f. Section 4). Although the notation in this paper allows for arbitrary values of L_k, in a practical system we will typically have $L_k \leq K_k$ to have an acceptable performance. The transmitted vector at the kth carrier after linear precoding is (see Figure 1(a)):

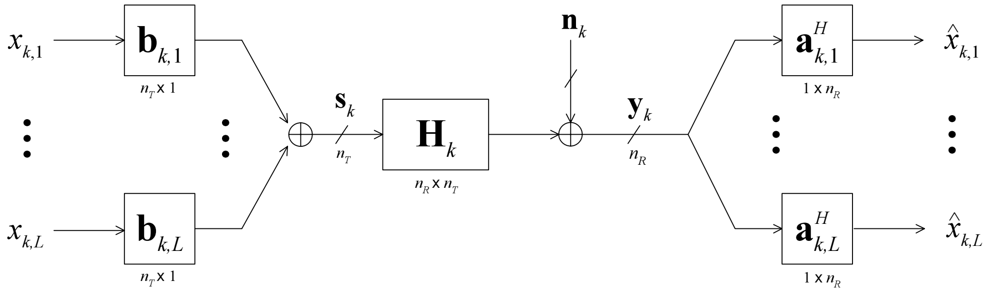
$$\mathbf{s}_k = \mathbf{B}_k \mathbf{x}_k = \sum_{i=1}^{L_k} \mathbf{b}_{k,i} x_{k,i} \tag{2}$$

where $\mathbf{x}_k \in \mathbb{C}^{L_k \times 1}$ represents the L_k transmitted symbols (we assume zero-mean unit-energy uncorrelated (white) symbols[1], *i.e.*, $\mathbb{E}\left[\mathbf{x}_k \mathbf{x}_k^H\right] = \mathbf{I}_{L_k}$), $\mathbf{B}_k \in \mathbb{C}^{n_T \times L_k}$ is the transmit matrix processing, $\mathbf{b}_{k,i} \triangleq \left[\mathbf{B}_k\right]_{:,i}$, and $x_{k,i} \triangleq \left[\mathbf{x}_k\right]_i$. We can think of each column of $\mathbf{B}_k$ as a different beamvector corresponding to each

[1] White symbols account, for example, to having independent bit streams. In case of having colored symbols due, for example, to a coded transmission, a prewhitening operation can be performed prior to precoding at the transmitter and the corresponding inverse operation can be performed after the equalizer at the receiver.



(a) Matrix processing interpretation at each carrier k.



(b) Multiple beamforming interpretation at each carrier k.

Figure 1: Matrix processing and multiple beamforming interpretations of the communication system (we assume for the clarity of the figure that $L_k = L \ \forall k$).

transmitted symbol, giving rise to a multiple beamforming architecture (see Figure 1(b)). Note that if only one symbol is transmitted per carrier ($L_k = 1 \ \forall k$), then (2) reduces to a classical beamforming structure with a single beamvector: $\mathbf{s}_k = \mathbf{b}_k x_k$. The transmitter is constrained in its average total transmit power[2]:

$$\sum_{k=1}^{N} \mathbb{E}\left[\, \|\mathbf{B}_k \mathbf{x}_k\|^2 \right] = \sum_{k=1}^{N} \|\mathbf{B}_k\|_F^2 \leq P_T \tag{3}$$

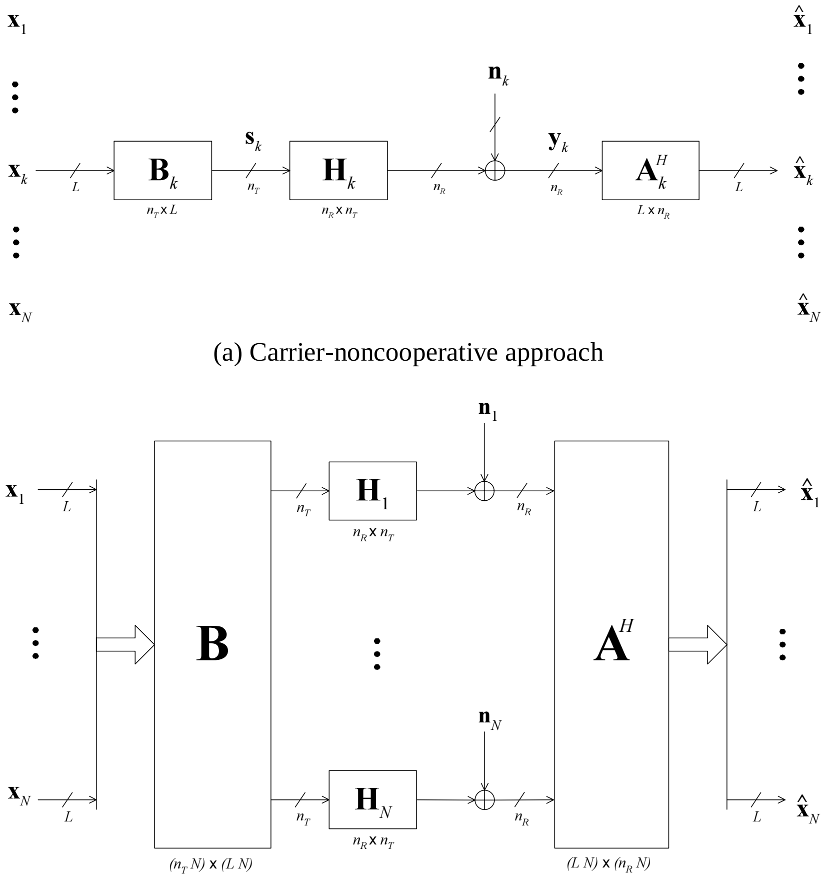
where P_T is the power in units of energy per block-transmission (or, equivalently, per OFDM symbol). The power in units of energy per symbol period is given by $P_s = P_T/N$ and the power in units of energy per second is P_s/T_s where T_s is the symbol period. In Section 6, a separate power constraint per antenna is considered. Note that a power constraint per carrier $\|\mathbf{B}_k\|_F^2 \leq P_k$ can be readily incorporated into the problem formulation.

The received vector at the kth carrier after the equalizer is

$$\hat{\mathbf{x}}_k = \mathbf{A}_k^H \mathbf{y}_k \tag{4}$$

where $\mathbf{A}_k^H \in \mathbb{C}^{L_k \times n_R}$ is the receive matrix processing and $\hat{\mathbf{x}}_k \in \mathbb{C}^{L_k \times 1}$ is the estimation of $\mathbf{x}_k$. Again, each column of $\mathbf{A}_k$ can be interpreted as a beamvector adapted to each spatial channel substream at carrier k, *i.e.*, $\hat{x}_{k,i} = \mathbf{a}_{k,i}^H \mathbf{y}_k$ (see Figure 1(b)).

[2]Equation (3) is a short-term power constraint (for each channel state) as opposed to a less restrictive long-term power constraint that would allow the transmit power to exceed P_T for some channel states as long as it is compensated by some other channel states (this constraint, however, requires knowledge of the channel statistics or at least of some future realizations of the channel).



(a) Carrier-noncooperative approach

(b) Carrier-cooperative approach

Figure 2: Carrier-cooperative vs. carrier-noncooperative approaches (we assume for the clarity of the figure that $L_k = L \ \forall k$).

Hitherto, only an independent processing at each carrier has been considered which we call *carrier-noncooperative approach* (see Figure 2(a)). This scheme, however, can be further generalized by allowing cooperation among carriers which we term *carrier-cooperative approach* (see Figure 2(b)). The signal model is obtained (similarly to (2)-(4)) by stacking the vectors corresponding to all carriers (*e.g.*, $\mathbf{x}^T = \left[\mathbf{x}_1^T, \cdots, \mathbf{x}_N^T\right]$), by considering global transmit and receive matrices $\mathbf{B} \in \mathbb{C}^{(n_T N) \times L_T}$ (the transmit power constraint reduces to $\|\mathbf{B}\|_F^2 \leq P_T$) and $\mathbf{A}^H \in \mathbb{C}^{L_T \times (n_R N)}$ where $L_T = \sum_{k=1}^N L_k$ is the total number of transmitted symbols, and by defining the global channel as $\mathbf{H} = \text{diag}\left(\{\mathbf{H}_k\}\right) \in \mathbb{C}^{(n_R N) \times (n_T N)}$. This general block processing scheme was used in [2] to obtain a capacity-achieving system. This model can easily cope with intermodulation terms unlike the noncooperative model of (1) that implicitly assumes the carriers completely orthogonal. The carrier-noncooperative processing model (2)-(4) can be obtained from the more general carrier-cooperative model by setting $\mathbf{B} = \text{diag}\left(\{\mathbf{B}_k\}\right)$ and $\mathbf{A} = \text{diag}\left(\{\mathbf{A}_k\}\right)$, *i.e.*, by imposing a block-diagonal structure on $\mathbf{B}$ and $\mathbf{A}$. In fact, it is this block-diagonal structure what makes the carrier-noncooperative scheme less general and, therefore, with a worse performance than the carrier-cooperative one. From an intuitive point of view, the reason of why this generalized model has a potential better performance is that it can reallocate the symbols among the carriers in an intelligent way (*e.g.*, if one carrier is in a deep fading, it will try to use other carriers instead), whereas the noncooperative scheme will always transmit L_k symbols through the kth carrier no matter the fading state of the carriers. From a mathematical point of view, however, the carrier-noncooperative model is more general since the carrier-cooperative scheme is obtained by particularizing $N = 1$ (a single carrier). Thus, in the sequel, the carrier-noncooperative matrix signal model is considered without loss of generality (w.l.o.g.).

4 Optimality of the Channel-Diagonalizing Structure

The joint transmit-receive matrix design is in general a complicated nonconvex problem. As previously mentioned, for some specific design criteria, the original complicated problem is greatly simplified because the channel turns out to be diagonalized by the transmit-receive processing, which allows a *scalarization* of the problem (meaning that all matrix equations are substituted with scalar ones). Examples are the minimization of the (weighted) sum of the MSE's of all channel spatial substreams [13, 3, 14], the minimization of the determinant of the MSE matrix [15], and the maximization of the mutual information [6, 2, 7]. Recall that for other interesting design criteria (such as the minimization of the average/maximum BER or the maximization of the minimum SINR) it is unknown whether the channel-diagonalizing structure is optimal.

In the following, we generalize these results by developing a unified framework. Instead of analyzing each design criterion in a separate way, we consider that the design is based on the minimization of some arbitrary objective function of the MSE's of all channel substreams $f_0(\{\text{MSE}_{k,i}\})$ where $\text{MSE}_{k,i}$ is the MSE of the ith spatial substream at the kth carrier (objective functions of the SINR's and of the BER's are readily incorporated as we show next). In particular, we will obtain that for a wide family of functions (Schur-concave and Schur-convex functions), the channel matrix is either fully diagonalized or diagonalized up to a very specific rotation of the data symbols.

The objective function f_0 is an indicator of how well the system performs. As an example, if two MIMO systems are identical except in one of the substreams for which one of the systems outperforms the other, any reasonable function f_0 should properly reflect this difference. Therefore, it suffices to consider only these *reasonable functions*[3]. Mathematically, this is equivalent to saying that the objective function f_0 must be increasing in each one of its arguments while having the rest fixed.

4.1 Optimum Receive Matrix

To design the system, we first easily derive the optimum receive matrices $\mathbf{A}_k$'s assuming the transmit ones $\mathbf{B}_k$'s fixed and then deal with the difficult part which is the derivation of the optimal transmit matrices $\mathbf{B}_k$'s (this two-step derivation has been independently used in [16]). The MSE matrix at the kth carrier is defined as the covariance matrix of the error vector (given by $\mathbf{e}_k \triangleq \hat{\mathbf{x}}_k - \mathbf{x}_k$):

$$\begin{aligned}\mathbf{E}_k(\mathbf{B}_k, \mathbf{A}_k) &\triangleq \mathbb{E}\left[(\hat{\mathbf{x}}_k - \mathbf{x}_k)(\hat{\mathbf{x}}_k - \mathbf{x}_k)^H\right] \\ &= \mathbf{A}_k^H \mathbf{R}_{y_k} \mathbf{A}_k + \mathbf{I} - \mathbf{A}_k^H \mathbf{H}_k \mathbf{B}_k - \mathbf{B}_k^H \mathbf{H}_k^H \mathbf{A}_k \end{aligned} \qquad (5)$$

[3]Given an *unreasonable* objective function, it is always possible to redefine it in a *reasonable* way so that it better reflects the system performance.

where $\mathbf{R}_{y_k} \triangleq \mathbb{E}\left[\mathbf{y}_k\mathbf{y}_k^H\right] = \mathbf{H}_k\mathbf{B}_k\mathbf{B}_k^H\mathbf{H}_k^H + \mathbf{R}_{n_k}$. The MSE of the ($k$th, ith) substream is the ith diagonal element of $\mathbf{E}_k$, *i.e.*,

$$\mathrm{MSE}_{k,i}\left(\mathbf{B}_k, \mathbf{a}_{k,i}\right) = \left[\mathbf{E}_k\right]_{ii} = \mathbf{a}_{k,i}^H\mathbf{R}_{y_k}\mathbf{a}_{k,i} + 1 - \mathbf{a}_{k,i}^H\mathbf{H}_k\mathbf{b}_{k,i} - \mathbf{b}_{k,i}^H\mathbf{H}_k^H\mathbf{a}_{k,i} \tag{6}$$

where $\mathbf{a}_{k,i}$ (resp. $\mathbf{b}_{k,i}$) is the ith column of $\mathbf{A}_k$ (resp. $\mathbf{B}_k$). Expression (6) is mathematically intractable since it is nonconvex in $(\mathbf{B}_k, \mathbf{a}_{k,i})$. However, for a given $\mathbf{B}_k$, $\mathrm{MSE}_{k,i}$ is convex in $\mathbf{a}_{k,i}$ and also independent of the other columns of $\mathbf{A}_k$ and of the other carriers, which means that each $\mathbf{a}_{k,i}$ can be independently optimized. To obtain the optimal receive matrix $\mathbf{A}_k^{\mathrm{opt}}$ in a more direct way, it suffices to find $\mathbf{A}_k$ such that the diagonal elements of $\mathbf{E}_k$ are minimized. This can be done regardless of the specific choice of the objective function f_0 since we know it is increasing in each argument. Alternatively, we can obtain $\mathbf{A}_k^{\mathrm{opt}}$ so that $\mathbf{E}_k(\mathbf{B}_k, \mathbf{A}_k^{\mathrm{opt}}) \leq \mathbf{E}_k\left(\mathbf{B}_k, \mathbf{A}_k\right)$ which in particular implies that the diagonal elements are minimized (in fact, both criteria are equivalent as shown in [29]). In other words, we want to solve:

$$\min_{\mathbf{A}_k^*} \quad \mathbf{c}^H\mathbf{E}_k\left(\mathbf{B}_k, \mathbf{A}_k\right)\mathbf{c} \qquad \forall \mathbf{c}.$$

Setting the gradient of $\mathbf{c}^H\mathbf{E}_k\mathbf{c} = \mathrm{Tr}\left(\mathbf{E}_k\mathbf{c}\mathbf{c}^H\right)$ to zero,

$$\nabla_{\mathbf{A}_k^*}\mathrm{Tr}\left(\mathbf{E}_k\mathbf{c}\mathbf{c}^H\right) = \mathbf{R}_{y_k}\mathbf{A}_k\mathbf{c}\mathbf{c}^H - \mathbf{H}_k\mathbf{B}_k\mathbf{c}\mathbf{c}^H = \mathbf{0} \qquad \forall \mathbf{c},$$

and particularizing $\mathbf{c}$ for all the vectors of the canonical base, it follows that

$$\mathbf{A}_k^{\mathrm{opt}} = \left(\mathbf{H}_k\mathbf{B}_k\mathbf{B}_k^H\mathbf{H}_k^H + \mathbf{R}_{n_k}\right)^{-1}\mathbf{H}_k\mathbf{B}_k. \tag{7}$$

Expression (7) is the linear minimum MSE (LMMSE) receiver or *Wiener filter* [30]. Using the optimal receive matrix $\mathbf{A}_k^{\mathrm{opt}}$, we obtain the following concentrated error matrix:

$$\begin{aligned}\mathbf{E}_k\left(\mathbf{B}_k\right) &\triangleq \mathbf{E}_k(\mathbf{B}_k, \mathbf{A}_k^{\mathrm{opt}}) \\ &= \mathbf{I} - \mathbf{B}_k^H\mathbf{H}_k^H\left(\mathbf{H}_k\mathbf{B}_k\mathbf{B}_k^H\mathbf{H}_k^H + \mathbf{R}_{n_k}\right)^{-1}\mathbf{H}_k\mathbf{B}_k \\ &= \left(\mathbf{I} + \mathbf{B}_k^H\mathbf{R}_{H_k}\mathbf{B}_k\right)^{-1}\end{aligned} \tag{8}$$

where we have used the matrix inversion lemma[4] and we have defined $\mathbf{R}_{H_k} \triangleq \mathbf{H}_k^H\mathbf{R}_{n_k}^{-1}\mathbf{H}_k$ (note that the eigenvectors and eigenvalues of $\mathbf{R}_{H_k}$ are the right singular vectors and the squared singular values respectively of the whitened channel $\mathbf{R}_{n_k}^{-1/2}\mathbf{H}_k$).

However, many objective functions are naturally expressed as functions of the SINR of each substream. The SINR at the kth carrier and the ith spatial substream is

$$\mathrm{SINR}_{k,i} \triangleq \frac{|\mathbf{a}_{k,i}^H\mathbf{H}_k\mathbf{b}_{k,i}|^2}{\mathbf{a}_{k,i}^H\mathbf{R}_{k,i}\mathbf{a}_{k,i}} \leq \mathbf{b}_{k,i}^H\mathbf{H}_k^H\mathbf{R}_{k,i}^{-1}\mathbf{H}_k\mathbf{b}_{k,i} \tag{9}$$

where $\mathbf{R}_{k,i} \triangleq \mathbf{H}_k\mathbf{B}_k\mathbf{B}_k^H\mathbf{H}_k^H + \mathbf{R}_{n_k} - \mathbf{H}_k\mathbf{b}_{k,i}\mathbf{b}_{k,i}^H\mathbf{H}_k^H$ is the interference-plus-noise covariance matrix seen by the (kth, ith) substream, the inequality comes from the Cauchy-Schwarz's inequality [31] (with vectors $(\mathbf{R}_{k,i}^{-1/2}\mathbf{H}_k\mathbf{b}_{k,i})$ and $(\mathbf{R}_{k,i}^{1/2}\mathbf{a}_{k,i})$), and the upper bound is achieved by $\mathbf{a}_{k,i} \propto \mathbf{R}_{k,i}^{-1}\mathbf{H}_k\mathbf{b}_{k,i} \propto \mathbf{R}_{y_k}^{-1}\mathbf{H}_k\mathbf{b}_{k,i}$, *i.e.*, the Wiener filter again. Noting that the MSE can be expressed as

$$\mathrm{MSE}_{k,i} = \left[\left(\mathbf{I} + \mathbf{B}_k^H\mathbf{H}_k^H\mathbf{R}_{n_k}^{-1}\mathbf{H}_k\mathbf{B}_k\right)^{-1}\right]_{ii} = \frac{1}{1 + \mathbf{b}_{k,i}^H\mathbf{H}_k^H\mathbf{R}_{k,i}^{-1}\mathbf{H}_k\mathbf{b}_{k,i}}, \tag{10}$$

[4] Matrix Inversion Lemma: $\left(\mathbf{A} + \mathbf{BCD}\right)^{-1} = \mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{B}\left(\mathbf{D}\mathbf{A}^{-1}\mathbf{B} + \mathbf{C}^{-1}\right)^{-1}\mathbf{D}\mathbf{A}^{-1}$.

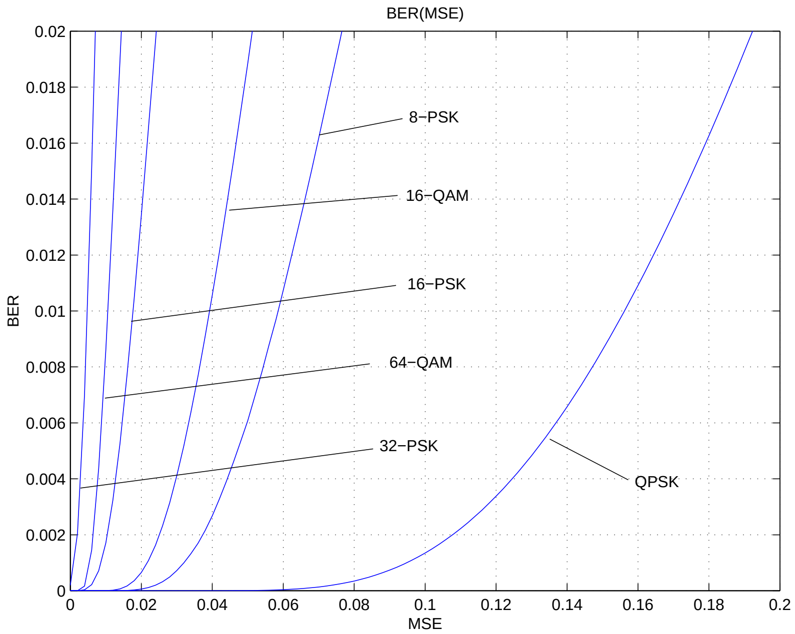


Figure 3: Convexity of the BER as a function of the MSE for the range of BER $\leq 2 \times 10^{-2}$.

the SINR can be easily related to the MSE as[5]

$$\mathrm{SINR}_{k,i} = \frac{1}{\mathrm{MSE}_{k,i}} - 1. \tag{11}$$

Maximizing the SINR is clearly equivalent to minimizing the MSE.

The performance of a digital communication system is ultimately given by the fraction of bits in error or bit error rate (BER). Under the Gaussian assumption, the symbol error probability P_e can be analytically expressed as a function of the SINR [32]:

$$P_e\left(\mathrm{SINR}\right) = \alpha\, \mathcal{Q}\left(\sqrt{\beta\ \mathrm{SINR}}\right) \tag{12}$$

where α and β are constants that depend on the signal constellation and $\mathcal{Q}$ is the $\mathcal{Q}$-function defined as $\mathcal{Q}\left(x\right) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\lambda^2/2} d\lambda$ [32]. It is sometimes convenient to use the Chernoff upper bound of the tail of the Gaussian distribution function $\mathcal{Q}\left(x\right) \leq \frac{1}{2} e^{-x^2/2}$ [33] to approximate the symbol error probability as $P_e \approx \frac{1}{2} \alpha\, e^{-\beta/2\ \mathrm{SINR}}$ (which becomes a good approximation for high values of the SINR). The BER can be approximately obtained from the symbol error probability (assuming that a Gray encoding is used to map the bits into the constellation points) as

$$\mathrm{BER} \approx P_e/k \tag{13}$$

where $k = \log_2 M$ is the number of bits per symbol and M is the constellation size.

It is important to remark that both the exact BER function and the Chernoff upper bound are convex decreasing functions of the SINR (see Appendix H). In addition, they are also convex increasing functions of the MSE for sufficiently small values of the argument (interestingly, for BPSK and QPSK

[5]Note that $0 < \mathrm{MSE} \leq 1$.

constellations this is true for any value of the argument) as can be observed from Figure 3 (see Appendix H for a formal proof). Note that minimizing the BER is tantamount to minimizing the MSE and to maximizing the SINR. As a rule-of-thumb, the exact BER function and the Chernoff upper bound are indeed convex in the MSE for a BER less than 2×10^{-2}. Note that this is a mild assumption, since practical systems have in general an uncoded BER[6] less than 2×10^{-2}. Therefore, for practical purposes, we can assume the exact BER and the Chernoff upper bound as convex functions of the MSE.

Summarizing, the Wiener filter has been obtained as the optimum linear receiver in the sense that minimizes each of the MSE's, maximizes each of the SINR's, and minimizes each of the BER's (in terms of capacity, the Wiener filter is capacity-lossless and simplifies the signal model). In addition, noting that the SINR can be expressed as a function of the MSE by (11) and that the BER can be expressed as a function of the SINR by (12)-(13), it suffices to focus on objective functions of the MSE's without loss of generality.

4.2 Optimum Transmit Matrix

To obtain the set of transmit matrices $\{\mathbf{B}_k\}$, we now consider the minimization of an arbitrary objective function of the diagonal elements of (8). As we now show, for Schur-concave and Schur-convex objective functions, the problem is scalarized and simplified (see Figure 4). In particular, the complicated nonconvex matrix function $\left[\left(\mathbf{I}+\mathbf{B}_k^H\mathbf{R}_{H_k}\mathbf{B}_k\right)^{-1}\right]_{ii}$ is simplified into a set of simple decoupled scalar expressions. We first consider the single-carrier case and then extend the results to the multicarrier case.

Theorem 1 *Consider the following constrained optimization problem:*

$$\begin{array}{ll} \min\limits_{\mathbf{B}} & f_0\left(\mathbf{d}(\mathbf{E}\left(\mathbf{B}\right))\right) \\ s.t. & \mathrm{Tr}\left(\mathbf{B}\mathbf{B}^H\right) \leq P_T \end{array}$$

where matrix $\mathbf{B} \in \mathbb{C}^{n_T \times L}$ *is the optimization variable,* $\mathbf{d}(\mathbf{E}\left(\mathbf{B}\right))$ *is the vector of diagonal elements of the MSE matrix* $\mathbf{E}\left(\mathbf{B}\right) = \left(\mathbf{I}+\mathbf{B}^H\mathbf{R}_H\mathbf{B}\right)^{-1}$ *(the diagonal elements of* $\mathbf{E}\left(\mathbf{B}\right)$ *are assumed in decreasing order w.l.o.g.),* $\mathbf{R}_H \in \mathbb{C}^{n_T \times n_T}$ *is a positive semidefinite Hermitian matrix, and* $f_0 : \mathbb{R}^L \longrightarrow \mathbb{R}$ *is an arbitrary objective function (increasing in each variable). It then follows that there is an optimal solution* $\mathbf{B}$ *of at most rank* $\check{L} \triangleq \min\left(L, \mathrm{rank}\left(\mathbf{R}_H\right)\right)$ *with the following structure:*

- *If* f_0 *is Schur-concave, then*

$$\mathbf{B} = \mathbf{U}_{H,1}\mathbf{\Sigma}_{B,1} \tag{14}$$

  where $\mathbf{U}_{H,1} \in \mathbb{C}^{n_T \times \check{L}}$ *has as columns the eigenvectors of* $\mathbf{R}_H$ *corresponding to the* $\check{L}$ *largest eigenvalues in increasing order and* $\mathbf{\Sigma}_{B,1} = [\mathbf{0}\ \mathrm{diag}\left(\{\sigma_{B,i}\}\right)] \in \mathbb{C}^{\check{L} \times L}$ *has zero elements except along the rightmost main diagonal (which can be assumed real w.l.o.g.).*

- *If* f_0 *is Schur-convex, then*

$$\mathbf{B} = \mathbf{U}_{H,1}\mathbf{\Sigma}_{B,1}\mathbf{V}_B^H \tag{15}$$

[6] Given an uncoded bit error probability of at most 10^{-2} and using a proper coding scheme, coded bit error probabilities with acceptable low values such as 10^{-6} can be obtained.

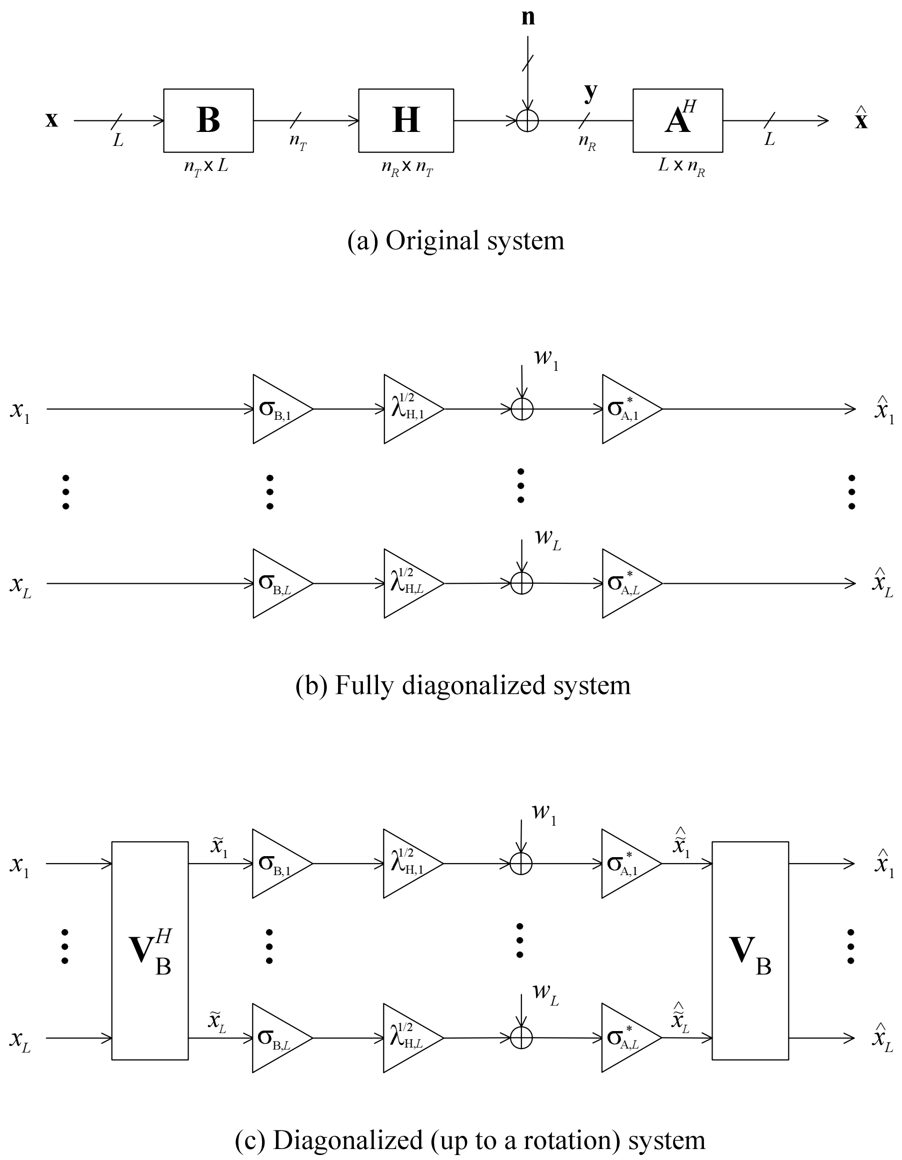


Figure 4: Original matrix system, fully diagonalized system, and diagonalized (up to a rotation) system.

where $\mathbf{U}_{H,1}$ *and* $\mathbf{\Sigma}_{B,1}$ *are defined as before, and* $\mathbf{V}_B \in \mathbb{C}^{L\times L}$ *is a unitary matrix such that* $\left(\mathbf{I}+\mathbf{B}^H\mathbf{R}_H\mathbf{B}\right)^{-1}$ *has identical diagonal elements. This rotation can be computed using the algorithm given in [34, Section IV-A] or with any rotation matrix* $\mathbf{Q}$ *that satisfies* $|\mathbf{Q}_{ik}| = |\mathbf{Q}_{il}|, \forall i,k,l$ *such as the Discrete Fourier Transform (DFT) matrix or the Hadamard matrix (when the dimensions are appropriate such as a power of two [33, p. 66]).*

Proof. See Appendix A. ■

For the simple case in which only one symbol per carrier is transmitted at each transmission, *i.e.*, a single spatial eigenmode $L = 1$ is utilized, Theorem 1 simplifies and the diagonal structure simply means that the spatial subchannel (eigenmode) with highest gain is used [35, 18].

For Schur-concave objective functions, the global communication process including pre- and post-processing $\mathbf{A}^H\mathbf{H}\mathbf{B}$ is fully diagonalized (see Figure 4(b)) as well as the MSE matrix $\mathbf{E}$. Among the

L established substreams, only $\check{L}$ are associated to nonzero channel eigenvalues whereas the remainder $L_0 = L - \check{L}$ are associated to zero eigenvalues. The global communication process is[7]

$$\hat{\mathbf{x}} = \left(\mathbf{I} + \boldsymbol{\Sigma}_{B,1}^H \mathbf{D}_{H,1} \boldsymbol{\Sigma}_{B,1}\right)^{-1} \boldsymbol{\Sigma}_{B,1}^H \mathbf{D}_{H,1}^{1/2} \left(\mathbf{D}_{H,1}^{1/2} \boldsymbol{\Sigma}_{B,1} \mathbf{x} + \mathbf{w}\right)$$

or, equivalently,

$$\hat{x}_i = \begin{cases} 0 & 1 \leq i \leq L_0 \\ \frac{\sigma_{B,(i-L_0)}^2 \lambda_{H,(i-L_0)}}{1+\sigma_{B,(i-L_0)}^2 \lambda_{H,(i-L_0)}} x_i + \frac{\sigma_{B,(i-L_0)} \lambda_{H,(i-L_0)}^{1/2}}{1+\sigma_{B,(i-L_0)}^2 \lambda_{H,(i-L_0)}} w_i & L_0 < i \leq L \end{cases}$$

where $\mathbf{D}_{H,1} = \text{diag}\left(\{\lambda_{H,i}\}_{k=1}^{\check{L}}\right)$, the $\lambda_{H,i}$'s are the $\check{L}$ largest eigenvalues of $\mathbf{R}_H$ in increasing order, the $\sigma_{B,i}^2$'s represent the allocated power, and $\mathbf{w}$ is a normalized equivalent white noise. The MSE matrix is $\mathbf{E} = \left(\mathbf{I} + \boldsymbol{\Sigma}_{B,1}^H \mathbf{D}_{H,1} \boldsymbol{\Sigma}_{B,1}\right)^{-1}$ and the corresponding MSE's are given by

$$\text{MSE}_i = \begin{cases} 1 & 1 \leq i \leq L_0 \\ \frac{1}{1+\sigma_{B,(i-L_0)}^2 \lambda_{H,(i-L_0)}} & L_0 < i \leq L. \end{cases} \tag{16}$$

Similarly, the SINR's are given using (11) by

$$\text{SINR}_i = \begin{cases} 0 & 1 \leq i \leq L_0 \\ \sigma_{B,(i-L_0)}^2 \lambda_{H,(i-L_0)} & L_0 < i \leq L. \end{cases} \tag{17}$$

Note that if $L > \text{rank}(\mathbf{R}_H)$ (equivalently, $L_0 > 0$), then the L_0 substreams associated to zero eigenvalues have an MSE equal to 1 or a zero SINR (which implies a BER equal to 0.5). Therefore, for Schur-concave objective functions, a communication system should be designed such that $L \leq \text{rank}(\mathbf{R}_H)$ in order to have an acceptable performance.

For Schur-convex objective functions, the global communication process including pre- and post-processing $\mathbf{A}^H\mathbf{H}\mathbf{B}$ is diagonalized only up to a very specific rotation of the data symbols (see Figure 4(c)) and the MSE matrix $\mathbf{E}$ is nondiagonal with equal diagonal elements (equal MSE's). In particular, assuming a pre-rotation of the data symbols at the transmitter $\tilde{\mathbf{x}} = \mathbf{V}_B^H \mathbf{x}$ and a post-rotation of the estimates at the receiver $\tilde{\hat{\mathbf{x}}} = \mathbf{V}_B^H \hat{\mathbf{x}}$, the same diagonalizing results of Schur-concave functions apply (see Figure 4(c)). Since the diagonal elements of the MSE matrix $\mathbf{E} = \left(\mathbf{I} + \mathbf{B}^H \mathbf{R}_H \mathbf{B}\right)^{-1}$ are equal whenever the appropriate rotation is included, the MSE's are identical and given by

$$\text{MSE}_i = \frac{1}{L} \text{Tr}(\mathbf{E}) = \frac{1}{L} \left(L_0 + \sum_{j=1}^{\check{L}} \frac{1}{1 + \sigma_{B,j}^2 \lambda_{H,j}} \right) \qquad 1 \leq i \leq L. \tag{18}$$

Similarly, the SINR's are given using (11) by

$$\text{SINR}_i = \frac{L}{L_0 + \sum_{j=1}^{\check{L}} \frac{1}{1+\sigma_{B,j}^2 \lambda_{H,j}}} - 1 \qquad 1 \leq i \leq L. \tag{19}$$

Note that during the design process, the rotation matrix can be initially ignored since the minimization can be based directly on the MSE expression in (18). The rotation can be computed at a later stage of the design as explained in Theorem 1. Observe that for Schur-convex functions (unlike for Schur-concave

[7]Note that $\mathbf{A} = \left(\mathbf{H}\mathbf{B}\mathbf{B}^H\mathbf{H}^H + \mathbf{R}_n\right)^{-1} \mathbf{H}\mathbf{B} = \mathbf{R}_n^{-1}\mathbf{H}\mathbf{B}\left(\mathbf{I} + \mathbf{B}^H\mathbf{H}^H\mathbf{R}_n^{-1}\mathbf{H}\mathbf{B}\right)^{-1}$.

ones), it is possible to have $L > \text{rank}(\mathbf{R}_H)$ (equivalently, $L_0 > 0$) and still obtain an acceptable performance. This is because the L symbols are transmitted over the $\check{L}$ nonzero eigenvalues in a distributed way (as opposed to the parallel and independent transmission of the symbols for fully diagonalized systems).

In both cases of Schur-concave and Schur-convex objective functions, the expressions of the MSE's have been scalarized in the sense that the original complicated matrix expressions have been reduced to simple scalar expressions ((16) and (18)). For Schur-concave functions, the specific power distribution among the established substreams will depend on the particular objective function f_0. Interestingly, for Schur-convex functions, the power distribution is independent of the specific choice of f_0 since both the MSE expression in (18) and the rotation matrix to make the diagonal elements of the MSE matrix equal are independent of f_0.

It is worth pointing out that there is a set of functions that are both Schur-concave and Schur-convex such as $\text{Tr}(\mathbf{E})$. Such functions happen to be invariant with respect to post-rotations of $\mathbf{B}$ and vice-versa (this can be easily proved using the same ideas of the proof of Theorem 1).

Theorem 1 is easily extended to the multicarrier case as follows. For any carrier k, consider the matrices corresponding to the rest of the carriers $\{\mathbf{B}_l\}_{l \neq k}$ fixed, and Theorem 1 can be directly invoked to show the optimal structure for $\mathbf{B}_k$.

5 Joint Tx-Rx Beamforming Design: a Convex Optimization Approach

In this section, using the optimal receive matrix given by (7) and the unified framework obtained in Theorem 1, we systematically consider a variety of design criteria. The potential of the proposed framework is made evident by showing that a great variety of interesting and appealing objective functions are either Schur-concave or Schur-convex and thus Theorem 1 can be applied to scalarize and simplify the design. The aim of this section is to express each problem in convex form, so that the well-developed body of literature on convex optimization theory [19, 20, 21] can be used to obtain optimal solutions very efficiently in practice using, for example, interior-point methods (c.f. subsection 2.1). In fact, it is possible in many cases to obtain simple closed-form solutions by means of the KKT optimality conditions that can be efficiently implemented in practice (see [36] for simple practical implementation algorithms derived from the KKT conditions and also for more design criteria).

Some of the considered design criteria have also been used in [35, 18] for the simple case of single beamforming. For simplicity of notation we define $z_{k,i} \triangleq \sigma^2_{B_k,i}$ and $\lambda_{k,i} \triangleq \lambda_{H_k,i}$. (Note that for Schur-concave functions with $L_k > \text{rank}(\mathbf{R}_{H_k})$, the $L_k - \check{L}_k$ substreams associated to zero eigenvalues are simply ignored in the optimization process.)

5.1 MSE-based Criteria

In the following, we optimize the MSE's by minimizing the arithmetic, geometric, and maximum means of the MSE's. We also show the equivalence of the minimization of the geometric mean, the minimization of the determinant of the MSE matrix, and the maximization of the mutual information.

5.1.1 Minimization of the ARITH-MSE

The minimization of the (weighted) arithmetic mean of the MSE's (ARITH-MSE) was considered in [13, 3, 14]. We deal with the weighted version as was extended in [14] under the unified framework of Theorem 1. The objective function is

$$f_0\left(\{\text{MSE}_{k,i}\}\right) = \sum_{k,i} \left(w_{k,i}\ \text{MSE}_{k,i}\right). \tag{20}$$

Lemma 4 *The function $f_0\left(\{x_i\}\right) = \sum_i \left(w_i\, x_i\right)$ (assuming $x_i \geq x_{i+1}$) is minimized when the weights are in increasing order $w_i \leq w_{i+1}$ and it is then a Schur-concave function.*

Proof. See Appendix B. ■

By Lemma 4, the objective function (20) is Schur-concave on each carrier k. Therefore, by Theorem 1, the diagonal structure is optimal and the MSE's are given by (16). The problem in convex form (the objective is convex and the constraints linear) is[8]

$$\begin{array}{ll} \min\limits_{\{z_{k,i}\}} & \sum_{k,i} w_{k,i} \frac{1}{1+\lambda_{k,i} z_{k,i}} \\ \text{s.t.} & \sum_{k,i} z_{k,i} \leq P_T, \\ & z_{k,i} \geq 0 \qquad 1 \leq k \leq N, \quad 1 \leq i \leq \check{L}_k. \end{array} \tag{21}$$

This particular problem can be solved very efficiently because the solution has a nice water-filling interpretation (from the KKT optimality conditions):

$$z_{k,i} = \left(\mu^{-1/2}\ w_{k,i}^{1/2}\ \lambda_{k,i}^{-1/2} - \lambda_{k,i}^{-1}\right)^+ \tag{22}$$

where $\mu^{-1/2}$ is the *water-level* chosen to satisfy the power constraint with equality.

5.1.2 Minimization of the GEOM-MSE

The objective function corresponding to the minimization of the weighted geometric mean of the MSE's (GEOM-MSE) is

$$f_0\left(\{\text{MSE}_{k,i}\}\right) = \prod_{k,i} \left(\text{MSE}_{k,i}\right)^{w_{k,i}}. \tag{23}$$

Lemma 5 *The function $f_0\left(\{x_i\}\right) = \prod_i x_i^{w_i}$ (assuming $x_i \geq x_{i+1} > 0$) is minimized when the weights are in increasing order $w_i \leq w_{i+1}$ and it is then a Schur-concave function.*

Proof. See Appendix C. ■

By Lemma 5, the objective function (23) is Schur-concave on each carrier k. Therefore, by Theorem 1, the diagonal structure is optimal and the MSE's are given by (16). The problem in convex form (since the objective is log-convex, it is also convex [21]) is [8]

$$\begin{array}{ll} \min\limits_{\{z_{k,i}\}} & \prod_{k,i} \left(\frac{1}{1+\lambda_{k,i} z_{k,i}}\right)^{w_{k,i}} \\ \text{s.t.} & \sum_{k,i} z_{k,i} \leq P_T, \\ & z_{k,i} \geq 0 \qquad 1 \leq k \leq N, \quad 1 \leq i \leq \check{L}_k. \end{array} \tag{24}$$

[8]Note that it is not necessary to explicitly include the constraints corresponding to $\text{MSE}_{k,i} \geq \text{MSE}_{k,i+1}$ in the convex problem formulation since an optimal solution always satisfies them.

This problem also has a water-filling solution (from the KKT optimality conditions):

$$z_{k,i} = \left(\mu^{-1}\, w_{k,i} - \lambda_{k,i}^{-1}\right)^+ \tag{25}$$

where μ^{-1} is the water-level chosen to satisfy the power constraint with equality. Note that for $w_{k,i} = 1$, (25) becomes the classical capacity-achieving water-filling solution[9] [6, 2].

5.1.3 Minimization of $|\mathbf{E}|$

The minimization of the determinant of the MSE matrix was considered in [15]. We now show how this particular criterion is easily accommodated in our framework as a Schur-concave function of the diagonal elements of the MSE matrix $\mathbf{E}$. (For the carrier-noncooperative case, simply consider the global MSE matrix defined as $\mathbf{E} = \text{diag}\left(\mathbf{E}_1, \cdots, \mathbf{E}_N\right)$.)

Using the fact that $\mathbf{X} \geq \mathbf{Y} \Rightarrow |\mathbf{X}| \geq |\mathbf{Y}|$, it follows that $|\mathbf{E}|$ is minimized for the choice of the receive matrix given by (7). From (8), it is clear that $|\mathbf{E}|$ does not change if the transmit matrices $\mathbf{B}_k$'s are post-multiplied by a unitary matrix (a rotation). Therefore, we can always choose a rotation matrix so that $\mathbf{E}$ is diagonal without loss of optimality (as we already knew from [15]) and then

$$|\mathbf{E}| = \prod_j \lambda_j\left(\mathbf{E}\right) = \prod_j \left[\mathbf{E}\right]_{jj}. \tag{26}$$

Therefore, the minimization of $|\mathbf{E}|$ is equivalent to the minimization of the (unweighted) product of the MSE's as in subsection 5.1.2.

5.1.4 Maximization of Mutual Information

The maximization of the mutual information can be used to obtain a capacity-achieving solution [6]:

$$\max_{\mathbf{Q}} \quad I = \log\left|\mathbf{I} + \mathbf{R}_n^{-1}\mathbf{H}\mathbf{Q}\mathbf{H}^H\right| \tag{27}$$

where $\mathbf{Q}$ is the transmit covariance matrix. Using the fact that $|\mathbf{I} + \mathbf{X}\mathbf{Y}| = |\mathbf{I} + \mathbf{Y}\mathbf{X}|$ and that $\mathbf{Q} = \mathbf{B}\mathbf{B}^H$ (from (2)), the mutual information can be expressed (see [29] for detailed connections between the mutual information and the MSE matrix) as

$$I = -\log|\mathbf{E}| \tag{28}$$

and, therefore, the maximization of I is equivalent to the minimization of $|\mathbf{E}|$ treated in subsection 5.1.3.

Hence, the minimization of the unweighted product of the MSE's, the minimization of the determinant of the MSE matrix, and the maximization of the mutual information are all equivalent criteria with solution given by a channel-diagonalizing structure and the classical capacity-achieving water-filling for the power allocation:

$$z_{k,i} = \left(\mu^{-1} - \lambda_{k,i}^{-1}\right)^+. \tag{29}$$

[9] Under the constraint of using $\check{L}_k$ substreams on each carrier k.

5.1.5 Minimization of the MAX-MSE

In general, the overall performance (average BER) is dominated by the substream with highest MSE. It makes sense then to minimize the maximum of the MSE's (MAX-MSE) [37]. The objective function is

$$f_0\left(\{\mathrm{MSE}_{k,i}\}\right) = \max_{k,i} \{\mathrm{MSE}_{k,i}\}\,. \tag{30}$$

Lemma 6 *The function $f_0(\{x_i\}) = \max_i \{x_i\}$ is a Schur-convex function.*

Proof. See Appendix D. ■

By Lemma 6, the objective function (30) is Schur-convex on each carrier k. Therefore, by Theorem 1, the optimal solution has a nondiagonal MSE matrix $\mathbf{E}_k$ with equal diagonal elements given by (18) which have to be minimized (scalarized problem). Recall that, after minimizing the MSE's, it remains to obtain the optimal rotation matrices so that the diagonal elements of the MSE matrices $\mathbf{E}_k$'s are identical. The scalarized problem in convex form (the objective is linear and the constraints are all convex) is

$$\begin{array}{lll} \displaystyle\min_{t,\{z_{k,i}\}} & t & \\ \text{s.t.} & t \geq \frac{1}{L_k}\left(\left(L_k - \check{L}_k\right) + \sum_{i=1}^{\check{L}_k} \frac{1}{1+\lambda_{k,i} z_{k,i}}\right) & 1 \leq k \leq N, \\ & \sum_{k,i} z_{k,i} \leq P_T, & \\ & z_{k,i} \geq 0 & 1 \leq k \leq N, \quad 1 \leq i \leq \check{L}_k. \end{array} \tag{31}$$

This problem has a multi-level water-filling solution (from the KKT optimality conditions):

$$z_{k,i} = \left(\bar{\mu}_k^{1/2}\, \lambda_{k,i}^{-1/2} - \lambda_{k,i}^{-1}\right)^+ \tag{32}$$

where $\{\bar{\mu}_k^{1/2}\}$ are multiple water-levels chosen to satisfy the constraints on t and the power constraint all with equality. For the case of single beamforming (*i.e.*, $L_k = 1$), the solution simplifies into

$$z_k = \lambda_k^{-1} \frac{P_T}{\sum_l \lambda_l^{-1}} \tag{33}$$

as was obtained in [35, 18]. For the single-carrier case (or multicarrier cooperative approach), problem (31) simplifies to the minimization of the unweighted ARITH-MSE considered in subsection 5.1.1 with solution $z_i = \left(\mu^{-1/2}\, \lambda_i^{-1/2} - \lambda_i^{-1}\right)^+$.

5.2 SINR-based Criteria

In the following, we optimize the SINR's by maximizing the arithmetic, geometric, harmonic, and minimum means of the SINR's. We also consider the maximization of the product of the terms $(1+\mathrm{SINR}_{k,i})$ and its connection to the capacity-achieving solution. We can now define the objective function to minimize as a function of the SINR's $\tilde{f}_0(\{\mathrm{SINR}_{k,i}\}) = f_0(\{\mathrm{MSE}_{k,i}\})$. Since in Theorem 1 we assumed $\mathrm{MSE}_{k,i} \geq \mathrm{MSE}_{k,i+1}$, the SINR's are in increasing order $\mathrm{SINR}_{k,i} \leq \mathrm{SINR}_{k,i+1}$.

5.2.1 Maximization of the ARITH-SINR

The objective function to be minimized for the maximization of the (weighted) arithmetic mean of the SINR's (ARITH-SINR) is

$$\tilde{f}_0\left(\{\mathrm{SINR}_{k,i}\}\right) = -\sum_{k,i} \left(w_{k,i}\ \mathrm{SINR}_{k,i}\right) \tag{34}$$

which can be expressed as a function of the MSE's using (11) as

$$f_0\left(\{\mathrm{MSE}_{k,i}\}\right) = \tilde{f}_0(\{\mathrm{MSE}_{k,i}^{-1} - 1\}) = -\sum_{k,i} w_{k,i}\left(\mathrm{MSE}_{k,i}^{-1} - 1\right). \tag{35}$$

Lemma 7 *The function $f_0\left(\{x_i\}\right) = -\sum_i w_i \left(x_i^{-1} - 1\right)$ (assuming $x_i \geq x_{i+1} > 0$) is minimized when the weights are in increasing order $w_i \leq w_{i+1}$ and it is then a Schur-concave function.*

Proof. See Appendix E. ■

By Lemma 7, the objective function (35) is Schur-concave on each carrier k. Therefore, by Theorem 1, the diagonal structure is optimal and the SINR's are given by (17). The problem expressed in convex form (it is actually an LP since the objective and the constraints are all linear) is[10]

$$\begin{array}{ll} \max\limits_{\{z_{k,i}\}} & \sum_{k,i} w_{k,i}\, \lambda_{k,i}\, z_{k,i} \\ \text{s.t.} & \sum_{k,i} z_{k,i} \leq P_T, \\ & z_{k,i} \geq 0 \qquad 1 \leq k \leq N, \quad 1 \leq i \leq \check{L}_k. \end{array} \tag{36}$$

The optimal solution is to allocate all the available power to the substream with maximum weighted gain ($w_{k,i}\, \lambda_{k,i}$) (otherwise the objective value could be increased by transferring power from other substreams to this substream). Although this solution maximizes indeed the weighted sum of the SINR's, it is a terrible solution in practice due to the extremely poor spectral efficiency (only one substream would be conveying information). This criterion gives a pathological solution and should not be used.

5.2.2 Maximization of the GEOM-SINR

The objective function to be minimized for the maximization of the (weighted) geometric mean of the SINR's (GEOM-SINR) is

$$\tilde{f}_0\left(\{\mathrm{SINR}_{k,i}\}\right) = -\prod_{k,i}\left(\mathrm{SINR}_{k,i}\right)^{w_{k,i}} \tag{37}$$

which can be expressed as a function of the MSE's using (11) as

$$f_0\left(\{\mathrm{MSE}_{k,i}\}\right) = \tilde{f}_0(\{\mathrm{MSE}_{k,i}^{-1} - 1\}) = -\prod_{k,i}(\mathrm{MSE}_{k,i}^{-1} - 1)^{w_{k,i}}. \tag{38}$$

Note that the maximization of the product of the SINR's is equivalent to the maximization of the sum of the SINR's expressed in dB.

Lemma 8 *The function $f_0\left(\{x_i\}\right) = -\prod_i \left(x_i^{-1} - 1\right)^{w_i}$ (assuming $0.5 \geq x_i \geq x_{i+1} > 0$) is minimized when the weights are in increasing order $w_i \leq w_{i+1}$ and it is then a Schur-concave function.*

Proof. See Appendix F. ■

By Lemma 8, the objective function (38) is Schur-concave on each carrier k provided that $\mathrm{MSE}_{k,i} \leq 0.5$ $\forall k, i$ (this is a mild assumption since a MSE greater than 0.5 is unreasonable for a practical communication system). Therefore, by Theorem 1, the diagonal structure is optimal and the SINR's are given by (17).

[10]Note that it is not necessary to explicitly include the constraints corresponding to $\mathrm{SINR}_{k,i} \leq \mathrm{SINR}_{k,i+1}$ in the convex problem formulation since an optimal solution always satisfies them.

The problem expressed in convex form (the weighted geometric mean is a concave function[11] [20, 21]) is [10]

$$
\begin{array}{ll}
\max\limits_{\{z_{k,i}\}} & \prod_{k,i} \left(\lambda_{k,i}\, z_{k,i}\right)^{\tilde{w}_{k,i}} \\
\text{s.t.} & \sum_{k,i} z_{k,i} \leq P_T, \\
& z_{k,i} \geq 0 \qquad 1 \leq k \leq N, \quad 1 \leq i \leq \check{L}_k,
\end{array}
\tag{39}
$$

where $\tilde{w}_{k,i} = w_{k,i}/\left(\sum_{l,j} w_{l,j}\right)$ and it is assumed that $\lambda_{k,i} > 0 \ \forall k, i$ (otherwise the problem has trivial solution $z_{k,i} = 0 \ \forall k, i$). The solution is easily obtained from the KKT optimality conditions as

$$
z_{k,i} = \tilde{w}_{k,i}\, P_T. \tag{40}
$$

Particularizing for a uniform weighting $w_{k,i} = 1$, the problem reduces to the maximization of the geometric mean subject to the arithmetic mean:

$$
\begin{array}{ll}
\max\limits_{\{z_{k,i}\}} & \prod_{k,i} z_{k,i}^{1/\check{L}_T} \\
\text{s.t.} & 1/\check{L}_T \sum_{k,i} z_{k,i} \leq P_T/\check{L}_T, \\
& z_{k,i} \geq 0,
\end{array}
\tag{41}
$$

where $\check{L}_T \triangleq \sum_{k=1}^{N} \check{L}_k$. From the arithmetic-geometric mean inequality $\left(\prod_k x_k\right)^{1/N} \leq \frac{1}{N} \sum_k x_k$ (with equality if and only if $x_k = x_l \ \forall k, l$) [31], it follows that the optimal solution is a uniform power allocation:

$$
z_{k,i} = P_T/\check{L}_T. \tag{42}
$$

Note that the uniform power distribution is commonly used due to its simplicity, *e.g.*, [38].

5.2.3 Maximization of the HARM-SINR

The maximization of the harmonic mean of the SINR's (HARM-SINR) was considered in [35] for the case of single beamforming. Using the unified framework of Theorem 1, we can extend this result to the case of multiple beamforming. The objective function to be minimized is

$$
\tilde{f}_0\left(\{\text{SINR}_{k,i}\}\right) = \sum_{k,i} \frac{1}{\text{SINR}_{k,i}} \tag{43}
$$

which can be expressed as a function of the MSE's using (11) as

$$
f_0\left(\{\text{MSE}_{k,i}\}\right) = \sum_{k,i} \frac{\text{MSE}_{k,i}}{1 - \text{MSE}_{k,i}}. \tag{44}
$$

Lemma 9 *The function* $f_0\left(\{x_i\}\right) = \sum_i \frac{x_i}{1-x_i}$ *(for* $0 \leq x_i < 1$*) is a Schur-convex function.*

Proof. See Appendix G. ∎

[11]The concavity of the geometric mean is easily verified by showing that the Hessian matrix is positive semidefinite for positive values of the arguments. The extension to include boundary points (points with zero-valued arguments) is straightforward either by using a continuity argument to show that $f\left(\theta \mathbf{x} + (1-\theta)\mathbf{y}\right) \geq \theta f\left(\mathbf{x}\right) + (1-\theta) f\left(\mathbf{y}\right)$ for $0 \leq \theta \leq 1$ or by considering the epigraph of the function and using [19, Prop. 2.7.4].

By Lemma 9, the objective function (44) is Schur-convex on each carrier k. Therefore, by Theorem 1, the optimal solution has a nondiagonal MSE matrix $\mathbf{E}_k$ with equal diagonal elements given by (18) which have to be minimized. The scalarized problem in convex form is

$$\begin{array}{lll} \min\limits_{\{t_k\},\{z_{k,i}\}} & \sum_k \frac{t_k}{1-t_k} & \\ \text{s.t.} & 1 > t_k \geq \frac{1}{L_k}\left(\left(L_k - \check{L}_k\right) + \sum_{i=1}^{\check{L}_k} \frac{1}{1+\lambda_{k,i} z_{k,i}}\right) & 1 \leq k \leq N, \\ & \sum_{k,i} z_{k,i} \leq P_T, & \\ & z_{k,i} \geq 0 & 1 \leq k \leq N, \quad 1 \leq i \leq \check{L}_k. \end{array} \tag{45}$$

The problem has a multi-level water-filling solution

$$z_{k,i} = \left(\bar{\mu}_k^{1/2}\, \lambda_{k,i}^{-1/2} - \lambda_{k,i}^{-1}\right)^+ \tag{46}$$

where $\{\bar{\mu}_k^{1/2}\}$ are multiple water-levels chosen to satisfy the lower constraints on the t_k's and the power constraint all with equality, and also the constraint $\bar{\mu}_k^{1/2} = \nu \frac{L_k^{-1/2}}{1-t_k}$ where ν is a positive parameter [36]. For the case of single beamforming (*i.e.*, $L_k = 1$), the solution reduces to $z_k = \lambda_k^{-1/2} \frac{P_T}{\sum_l \lambda_l^{-1/2}}$ [35]. For the single-carrier case (or multicarrier cooperative approach), the problem simplifies to that considered in subsection 5.1.1.

5.2.4 Maximization of the MIN-SINR

The objective function to be minimized for the maximization of the minimum of the SINR's (MIN-SINR) is

$$\tilde{f}_0\left(\{\text{SINR}_{k,i}\}\right) = -\min_{k,i} \{\text{SINR}_{k,i}\}\,. \tag{47}$$

This design criterion is equivalent to the minimization of the maximum MSE treated with detail in subsection 5.1.5. In [14], the same criterion was used imposing a channel diagonal structure.

5.2.5 Maximization of the PROD-(1+SINR)

Consider for a moment the following maximization:

$$\max \quad \prod_{k,i} \left(1 + \text{SINR}_{k,i}\right). \tag{48}$$

Using the relation between the MSE and the SINR in (11), this maximization can be equivalently expressed as the minimization of $\prod_{k,i} \text{MSE}_{k,i}$ as in (23) with $w_{k,i} = 1$, as the minimization of the determinant of the MSE matrix (subsection 5.1.3), and as the maximization of the mutual information (subsection 5.1.4) with solution given by the capacity-achieving expression (29). This result is completely natural since maximizing the logarithm of (48) is tantamount to maximizing the mutual information $I = \sum_{k,i} \log\left(1 + \text{SINR}_{k,i}\right)$.

5.3 BER-based Criteria

Next, we consider the minimization of the average BER (the minimization of the maximum of the BER's (MAX-BER) is equivalent to the maximization of the minimum of the SINR's and to the minimization of the maximum of the MSE's provided that the same constellations are used on all the substreams).

5.3.1 Minimization of the ARITH-BER

The minimization of the average BER or of the arithmetic mean of the BER's (ARITH-BER) can be considered as the best criterion (assuming that after the linear processing at the receiver each substream is detected independently). In practice, multicarrier communication systems use some type of coding over the carriers and/or over different transmissions to reduce the BER (usually some orders of magnitude). The ultimate measure is then the coded BER as opposed to the uncoded BER (obtained without using any coding). However, the coded BER is strongly related to the uncoded BER (in fact, for codes based on hard decisions, both quantities are strictly related). In such cases, it suffices to focus on the uncoded BER when designing the uncoded part of a communication system.

In [39], the minimization of the average BER (and also of the Chernoff upper bound) is treated in detail when a diagonal structure is imposed. This design criterion has been independently considered in [40] under a ZF constraint obtaining a nondiagonal optimal MSE matrix (in agreement with our results). The objective function is

$$\tilde{f}_0\left(\{\mathrm{BER}_{k,i}\}\right) = \sum_{k,i} \mathrm{BER}_{k,i} \tag{49}$$

which can be expressed as a function of the MSE's using (11) and (12)-(13) as

$$f_0\left(\{\mathrm{MSE}_{k,i}\}\right) = \sum_{k,i} \mathrm{BER}\left(\mathrm{MSE}_{k,i}^{-1} - 1\right). \tag{50}$$

Lemma 10 *The function* $f_0\left(\{x_i\}\right) = \sum_i \mathrm{BER}\left(x_i^{-1} - 1\right)$ *(assuming* $\theta \geq x_i > 0$*, for sufficiently small* θ *such that* $\mathrm{BER}\left(x_i^{-1} - 1\right) \leq 2 \times 10^{-2}$*,* $\forall i$*) is a Schur-convex function.*

Proof. See Appendix I. ■

By Lemma 10, the objective function (50) is Schur-convex on each carrier k (assuming the same constellation/coding on all substreams of the kth carrier) provided that $\mathrm{BER}_{k,i} \leq 2 \times 10^{-2}$ (interestingly, for BPSK and QPSK constellations, this is true for any value of the BER). Therefore, by Theorem 1, the optimal solution has a nondiagonal MSE matrix $\mathbf{E}_k$ with equal diagonal elements given by (18) which have to be minimized. The scalarized problem in convex form is[12]

$$\begin{array}{lll}
\underset{\{t_k\},\{z_{k,i}\}}{\min} & \sum_k \alpha_k \mathcal{Q}\left(\sqrt{\beta_k \left(t_k^{-1} - 1\right)}\right) & \\
\text{s.t.} & \theta \geq t_k \geq \frac{1}{L_k}\left(\left(L_k - \check{L}_k\right) + \sum_{i=1}^{\check{L}_k} \frac{1}{1+\lambda_{k,i} z_{k,i}}\right) & 1 \leq k \leq N, \\
& \sum_{k,i} z_{k,i} \leq P_T, & \\
& z_{k,i} \geq 0, & 1 \leq k \leq N, \quad 1 \leq i \leq \check{L}_k.
\end{array} \tag{51}$$

Note that we have explicitly included the upper bound θ on the MSE's to guarantee the convexity of the BER function and therefore of the whole problem. For a general case with $L_k > 1$ and $N > 1$, problem (51) does not have a simple closed-form solution and one has to resort to general purpose iterative methods such as interior-point methods (see subsection 2.1). For the single-carrier case (or multicarrier cooperative approach), the problem simplifies to the ARITH-MSE criterion considered in subsection 5.1.1 plus the rotation matrix to make the diagonal elements of the MSE matrix equal.

[12] We have implicitly assumed for each carrier the same constellation and code on all the spatial eigenmodes.

5.4 Remarks

Some observations are in order:

- Most of the presented methods under the framework of convex optimization theory have nice closed-form solutions which can be easily implemented in practice (see [36] for simple practical algorithms derived from the KKT optimality conditions to implement the water-filling solutions).
- Method ARITH-BER is clearly the best in terms of average BER and is therefore considered as a benchmark. For the carrier-noncooperative approach, it does not have a closed-form solution and an iterative approach is necessary such as an interior-point method (see [21] for practical implementation details). Interestingly, for the single-carrier and multicarrier-cooperative approaches, the solution can be obtained in closed-form as mentioned below.
- Methods ARITH-MSE, HARM-SINR, and MAX-MSE have very simple solutions and, as will be observed in the simulations, perform really close to the benchmark given by ARITH-BER. These methods should therefore be considered for practical purposes.
- Two novel multi-level water-filling solutions have been obtained for the MAX-MSE and the HARM-SINR criteria (see [36] for practical implementation algorithms).
- Cooperation among carriers improves performance without significant increase on the complexity (each carrier can be diagonalized independently and then the largest eigenmodes are selected).
- A striking result (as mentioned in Section 4) is that for single-carrier and multicarrier-cooperative systems, all criteria with Schur-convex objective functions (*e.g.*, MAX-MSE, HARM-SINR, MIN-SINR, ARITH-BER[13] , and MAX-BER [13]) have the same optimal solution. Hence, the best performance (given by the ARITH-BER criterion with a carrier-cooperative approach) has a closed-form solution which can be obtained in practice with low complexity using the simple water-filling solution of the ARITH-MSE criterion in (22) plus the rotation matrix.
- It is very common in the literature of equalization to include a ZF constraint in the design. Such a constraint can be easily introduced in the unified framework (see [36] for details), although the performance degrades due to the additional constraint.

6 Introducing Additional Constraints

As explained in Section 2, one of the nice properties of expressing a problem in convex form is that additional constraints can be added as long as they are convex without affecting the solvability of the problem. Of course, with the additional constraints, the closed-form solutions previously obtained are not valid any more.

[13] Recall that with carrier cooperation, the ARITH-BER and MAX-BER methods require all spatial/carrier substreams to use the same constellation/coding scheme in order to be Schur-convex.

6.1 Dynamic Range of Power Amplifier

We can easily add constraints on the dynamic range of the power amplifier at each transmit antenna element as was done in [22]. Consider a Schur-concave objective function and assume for simplicity $L_k = L \; \forall k$. From the optimal structure in (14) $\mathbf{B}_k = \mathbf{U}_{H_k,1}\mathbf{\Sigma}_{B_k,1}$, the total average transmitted power (in units of energy per symbol period) by the ith antenna is

$$P_i = \frac{1}{N}\sum_{k=1}^{N}\sum_{l=1}^{L} \left| [\mathbf{B}_k]_{i,l} \right|^2 = \frac{1}{N}\sum_{k=1}^{N}\sum_{l=1}^{L} \sigma^2_{B_k,l} \left| [\mathbf{U}_{H_k,1}]_{i,l} \right|^2 \tag{52}$$

which is linear in the variables $\{\sigma^2_{B_k,i}\}$. (For the carrier-cooperative scheme, $P_i = \frac{1}{N}\sum_{k=1}^{N}\sum_{l=1}^{NL} \left| [\mathbf{B}]_{i+(k-1)n_T,l} \right|^2$.) Therefore, the following constraints are linear

$$\alpha_i^L \le P_i \le \alpha_i^U \qquad 1 \le i \le n_T$$

where α_i^L and α_i^U are the lower and upper bounds for the ith antenna. Similarly, it is straightforward to set limits on the relative dynamic range of a single element in comparison to the total power for the whole array [22]:

$$\rho_i^L \; P_{\text{array}} \le P_i \le \rho_i^U \; P_{\text{array}} \qquad 1 \le i \le n_T$$

where ρ_i^L and ρ_i^U are the relative bounds, and $P_{\text{array}} = \sum_{i=1}^{n_T} P_i$ is the total power also linear in $\{\sigma^2_{B_k,i}\}$.

6.2 Peak to Average Ratio (PAR)

One of the main practical problems that OFDM systems face is the PAR. Indeed, multicarrier signals exhibit Gaussian-like time-domain waveforms with relatively high PAR, *i.e.*, they exhibit large amplitude spikes when several frequency components add in-phase. These spikes may have a serious impact on the design complexity and feasibility of the transceiver's analog front-end (*i.e.*, high resolution of D/A-A/D converters and power amplifiers with a linear behavior over a large dynamical range). In practice, the transmitted signal has to be clipped when it exceeds a certain threshold which has detrimental effects on the BER. A variety of techniques have been devised to deal with the PAR [41, 42]. In this section we show how the PAR can be taken into account into the design of the beamvectors using a convex optimization framework. Note that the already existing techniques to cope with the PAR and this approach are not exclusive and can be simultaneously used.

The PAR is defined as

$$\text{PAR} \triangleq \max_{0 \le t \le T_s} \frac{A^2(t)}{\sigma^2} \tag{53}$$

where T_s is the symbol period, $A(t)$ is the zero-mean transmitted signal, and $\sigma^2 = \mathbb{E}\left[A^2(t)\right]$. Since the number of carriers is usually large ($N \ge 64$), $A(t)$ can be accurately modeled as a Gaussian random process (central-limit theorem) with zero mean and variance σ^2 [41]. Using this assumption, the probability that the PAR exceeds certain threshold or, equivalently, the probability that the instantaneous amplitude exceeds a clipping value A_{clip} is

$$\Pr\left\{|A(t)| > A_{\text{clip}}\right\} = 2\;\mathcal{Q}\left(\frac{A_{\text{clip}}}{\sigma}\right). \tag{54}$$

Delay (ns)	Average relative power (dB)
0	-3.3
10	-3.6
20	-3.9
30	-4.2
50	0.0
80	-0.9
110	-1.7
140	-2.6
180	-1.5
230	-3.0
280	-4.4
330	-5.9
400	-5.3
490	-7.9
600	-9.4
730	-13.2
880	-16.3
1050	-21.2

(a)

$$\mathbf{R}_{\mathrm{BS}} = \begin{bmatrix} 1.0000 & 0.4154 & 0.2057 & 0.1997 \\ 0.4154 & 1.0000 & 0.3336 & 0.3453 \\ 0.2057 & 0.3336 & 1.0000 & 0.5226 \\ 0.1997 & 0.3453 & 0.5226 & 1.0000 \end{bmatrix}$$

$$\mathbf{R}_{\mathrm{MS}} = \begin{bmatrix} 1.0000 & 0.3644 & 0.0685 & 0.3566 \\ 0.3644 & 1.0000 & 0.3245 & 0.1848 \\ 0.0685 & 0.3245 & 1.0000 & 0.3093 \\ 0.3566 & 0.1848 & 0.3093 & 1.0000 \end{bmatrix}$$

(b)

Figure 5: (a) Power delay profile type C for HIPERLAN/2. (b) Envelope correlation matrices at the base station (BS) and at the mobile station (MS) in the *Nokia* environment.

The clipping probability of an OFDM symbol is then [41]

$$P_{\text{clip}}(\sigma) = 1 - \left(1 - 2\,\mathcal{Q}\left(\frac{A_{\text{clip}}}{\sigma}\right)\right)^{2N}. \tag{55}$$

In other words, in order to have a clipping probability lower than P with respect to the maximum instantaneous amplitude A_{clip}, the average signal power must satisfy

$$\sigma \leq \sigma_{\text{clip}}(P) = \frac{A_{\text{clip}}}{\mathcal{Q}^{-1}\left(\frac{1-(1-P)^{1/(2N)}}{2}\right)}. \tag{56}$$

When using multiple antennas for transmission, the previous equation has to be satisfied for all transmit antennas. Those constraints can be easily incorporated in any of the convex designs derived in Section 5 with a Schur-concave objective function. Using (52) the constraint is

$$\frac{1}{N}\sum_{k=1}^{N}\sum_{l=1}^{L}\sigma_{B_k,l}^2\,|[\mathbf{U}_{H_k,1}]_{i,l}|^2 \leq \sigma_{\text{clip}}^2 \qquad 1 \leq i \leq n_T \tag{57}$$

which is linear in the optimization variables $\{\sigma_{B_k,i}^2\}$. Such a constraint has two effects in the solution: (i) the power distribution over the carriers changes with respect to the distribution without the constraint and (ii) the total transmitted power drops as necessary.

7 Simulation Results

For the numerical results, we have chosen the European standard HIPERLAN/2 for WLAN [9]. It is based on the multicarrier modulation OFDM (64 carriers are used in the simulations). We consider multiple antennas at both the transmitter and the receiver, obtaining therefore the multicarrier MIMO model used throughout the paper. Perfect CSI is assumed at both sides of the communication link[14].

[14] In practice, channel estimation errors exist and it is therefore necessary to quantify the loss for each of the methods. Alternatively, it is possible to take channel estimation errors into account in the design either from a worst-case or from a Bayesian perspective (c.f. [36]).

The frequency selectivity of the channel is modeled using the power delay profile type C for HIPERLAN/2 as specified in [43] (see Figure 5(a)), which corresponds to a typical large open space indoor environment for NLOS[15] conditions with 150ns average r.m.s. delay spread and 1050ns maximum delay (the sampling period is 50ns) [9]. The spatial correlation of the MIMO channel is modeled according to the *Nokia* model defined in [44] (which corresponds to a reception hall) specified by the correlation matrices of the envelope of the channel fading at the transmit and receive side given in Figure 5(b) where the base station is the receiver (uplink) (see [44] for details of the model). It provides a large open indoor environment with two floors, which could easily illustrate a conference hall or a shopping galleria scenario. The matrix channel generated was normalized so that $\sum_n \mathbb{E}\left[|\mathbf{H}_{ij}(n)|^2\right] = 1$. The SNR is defined as the transmitted power normalized with the noise variance.

For the numerical simulations, the following design criteria have been considered: ARITH-MSE, GEOM-MSE, MAX-MSE (equivalently, MIN-SINR or MAX-BER), GEOM-SINR, HARM-SINR, and ARITH-BER (benchmark). The utilization of the Chernoff upper bound instead of the exact BER function gives indistinguishable results and is therefore not presented in the simulation results. Unless otherwise specified, carrier-noncooperative approaches are considered. The performance is given in terms of outage BER (averaged over the channel substreams), *i.e.*, the BER that can be guaranteed with some probability or, equivalently, the BER that is not achieved with some small outage probability. In particular, we consider the BER with an outage probability of 5%. Note that for typical systems with delay constraints, the outage BER is a more realistic measure than the commonly used mean BER that only makes sense when the transmission coding block is long enough to reveal the long-term ergodic properties of the fading process (no delay constraints).

Single Beamforming

First we show some results when using a single channel spatial substream ($L_k = 1 \ \forall k$). In Figure 6, the BER is plotted vs. the SNR for a 2×2 MIMO channel using QPSK constellations. Clearly, the ARITH-BER criterion has the lowest BER because it was designed for that. The MAX-MSE criterion performs really close to the ARITH-BER and can be considered the second best criterion. The HARM-SINR and also the ARITH-MSE perform reasonably well (in fact, for values of the BER higher than 10^{-2} they outperform the MAX-MSE). The GEOM-MSE and the GEOM-SINR criteria perform really bad in terms of BER and should not be used. In Figure 7, the same results are shown for a 4×2 MIMO channel using 16-QAM constellations and the same observations hold.

Therefore, the best criteria are (in order): ARITH-BER, MAX-MSE, HARM-SINR, and ARITH-MSE.

Including PAR Constraints

We now consider the introduction of PAR constraints as described in Section 6. We parameterize the clipping amplitude with respect to μ as $A_{\text{clip}} = \mu\sqrt{\frac{P_T}{n_T}}$ to make the results independent of the total transmitted power P_T. In Figure 8, the probability of clipping along with the BER (when using QPSK constellations) is shown for the ARITH-MSE criterion in a 2×2 MIMO channel both with PAR constraints ($P_{\text{clip}} = 10^{-2}$) and without them. In 8(a), the results are shown as a function of μ. It can be observed how the design with the PAR constraints always has a clipping probability no greater than the prespecified

[15]NLOS: Non-line-of-sight.

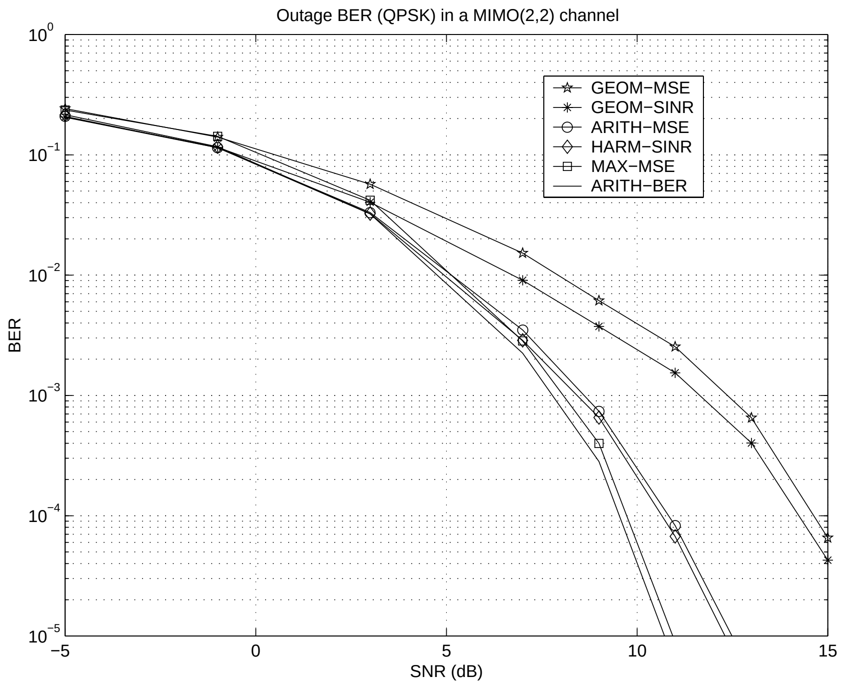


Figure 6: BER (at an outage probability of 5%) vs. SNR when using QPSK in a 2×2 MIMO channel with $L = 1$ for the GEOM-MSE, GEOM-SINR, ARITH-MSE, HARM-SINR, MAX-MSE, and ARITH-BER criteria (without carrier cooperation).

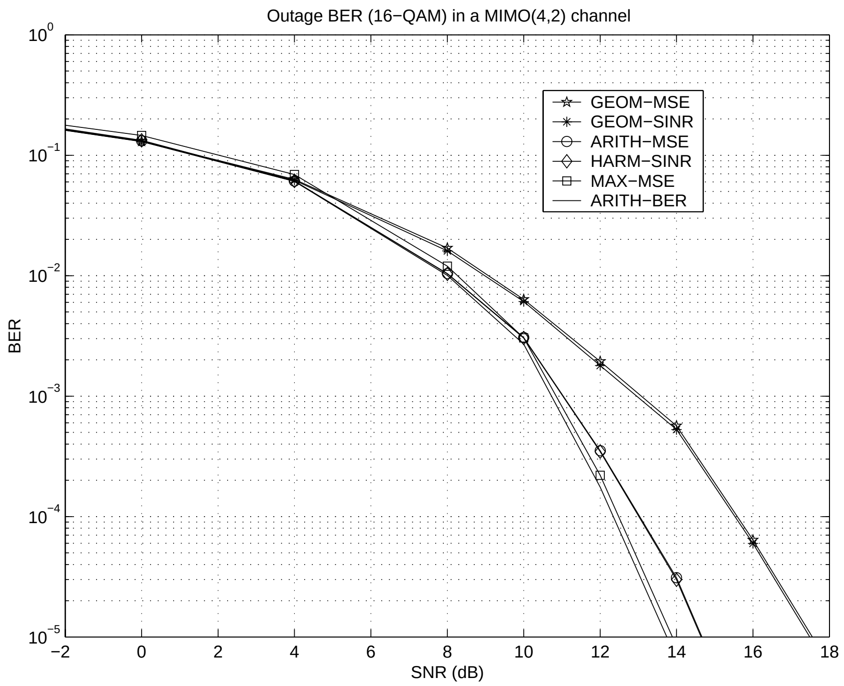
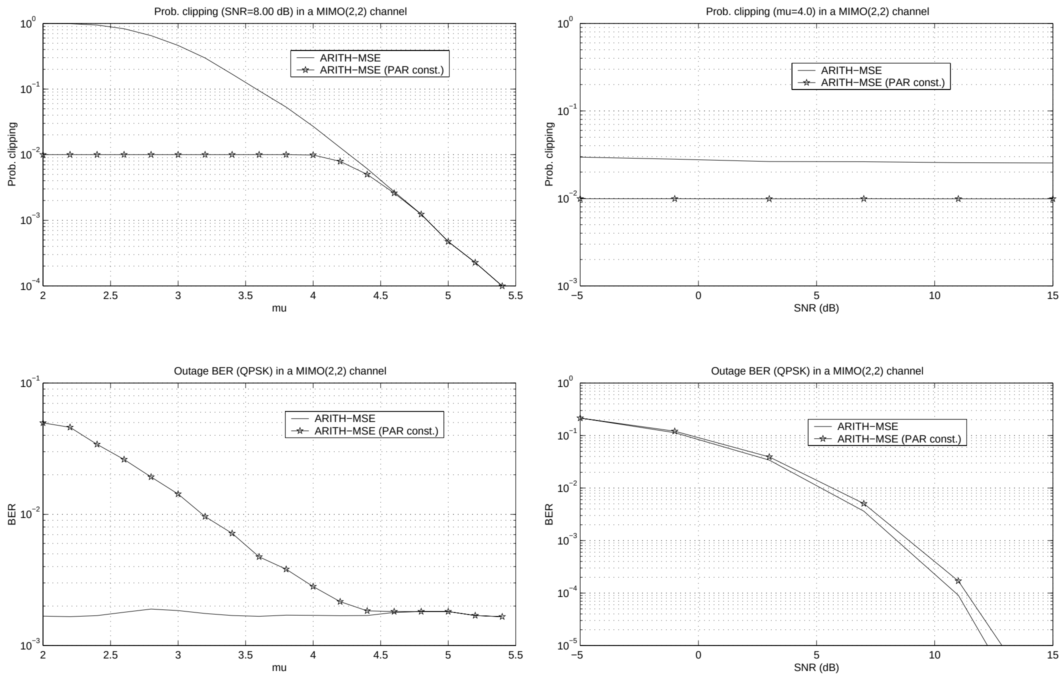


Figure 7: BER (at an outage probability of 5%) vs. SNR when using 16-QAM in a 4×2 MIMO channel (2 transmit and 4 receive antennas) with $L = 1$ for the GEOM-MSE, GEOM-SINR, ARITH-MSE, HARM-SINR, MAX-MSE, and ARITH-BER criteria (without carrier cooperation).



(a) (b)

Figure 8: Probability of clipping and BER (at an outage probability of 5%) when using QPSK in a 2×2 MIMO channel with $L = 1$ for the ARITH-MSE criterion with and without PAR constraints (without carrier cooperation): (a) as a function of μ (for SNR=8 dB and $P_{\text{clip}} \leq 10^{-2}$), and (b) as a function of the SNR (for $\mu = 4$ and $P_{\text{clip}} \leq 10^{-2}$).

value 10^{-2} as expected. The BER, however, can be severely affected if a very low clipping probability is imposed due to power back-offs. From Figure 8(a), a choice of $\mu = 4$ seems reasonable. In Figure 8(b), the results are shown as a function of the SNR for $\mu = 4$. For the design with PAR constraints, the BER is slightly higher due to the additional constraint. However, the system is guaranteed to have a clipping probability of at most 10^{-2} unlike in the unconstrained case where nothing can be guaranteed. Recall that in a practical system, the final BER increases due to the clipping.

Multiple Beamforming

We now consider the simultaneous transmission of more than one symbol per carrier, *i.e.*, multiple beamforming (we consider $L_k = L \ \forall k$).

In Figure 9, the BER is plotted vs. the SNR for a 4×4 MIMO channel with $L = 2$ using QPSK constellations. In general, similar observations hold as for the single beamforming case. However, it is worth pointing out that in this case the HARM-SINR method performs much closer to the benchmark that the ARITH-MSE method. For higher values of the ratio $L/\min(n_T, n_R)$, the superiority of Schur-convex criteria with respect to the Schur-concave methods (which have a channel-diagonalizing structure) becomes more clear (c.f. [36]).

Carrier Cooperation

We now analyze the improvement in performance when using cooperation among carriers for the best methods: ARITH-MSE, HARM-SINR, MAX-MSE, and ARITH-BER. Recall that with carrier cooperation, the HARM-SINR, MAX-MSE and ARITH-BER criteria provide the same solution since they are

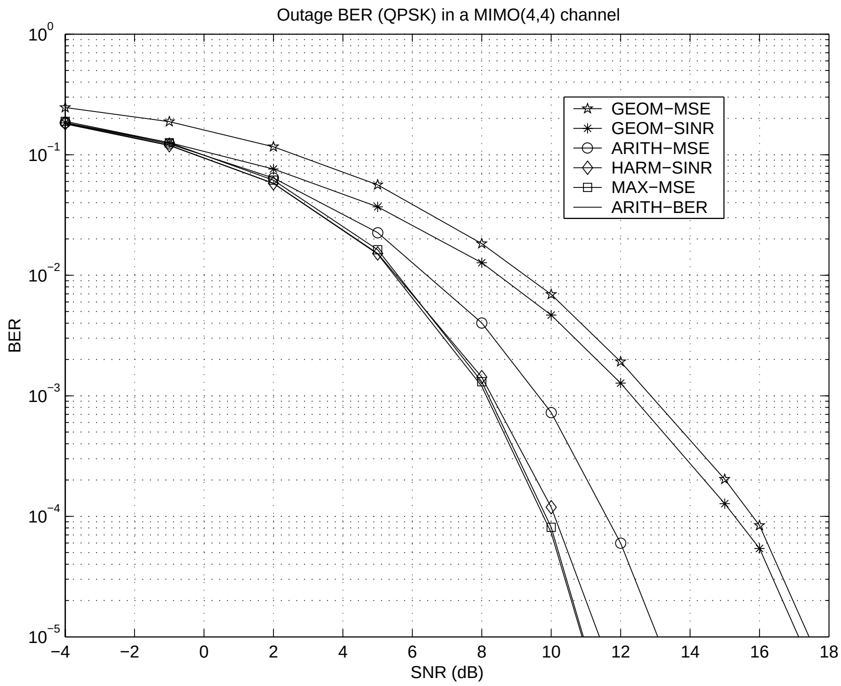


Figure 9: BER (at an outage probability of 5%) vs. SNR when using QPSK in a 4×4 MIMO channel with $L = 2$ for the GEOM-MSE, GEOM-SINR, ARITH-MSE, HARM-SINR, MAX-MSE, and ARITH-BER criteria (without carrier cooperation).

all Schur-convex functions (see subsection 5.4).

In Figure 10, the BER is plotted vs. the SNR with and without carrier cooperation for a 2×2 MIMO channel with $L_k = 1$ using QPSK constellations. In this case, carrier cooperation gives an improvement of $0.5 - 2$dB. For higher values of the ratio $L/\min(n_T, n_R)$ the improvement is even more significant (c.f. [36]).

8 Conclusions

In this paper, we have formulated and solved the joint design of transmit and receive multiple beamvectors or beam-matrices (also known as linear precoders and equalizers) for multicarrier MIMO systems under a variety of design criteria. Instead of considering each design criterion in a separate way, we have developed a unifying framework that generalizes the existing results by considering two families of objective functions that embrace most reasonable criteria to design a communication system: Schur-concave and Schur-convex functions. For Schur-concave objective functions, the channel-diagonalizing structure is always optimal, whereas for Schur-convex functions, an optimal solution diagonalizes the channel only after a very specific rotation of the transmitted symbols.

Once the optimal structure of the communication process has been obtained, the design problem has been formulated within the powerful framework of convex optimization theory, in which a great number of interesting design criteria can be easily accommodated and efficiently solved even though closed-form expressions may not exist. From this perspective, a variety of design criteria have been analyzed and, in particular, optimal beamvectors have been derived in the sense of having minimum average BER.

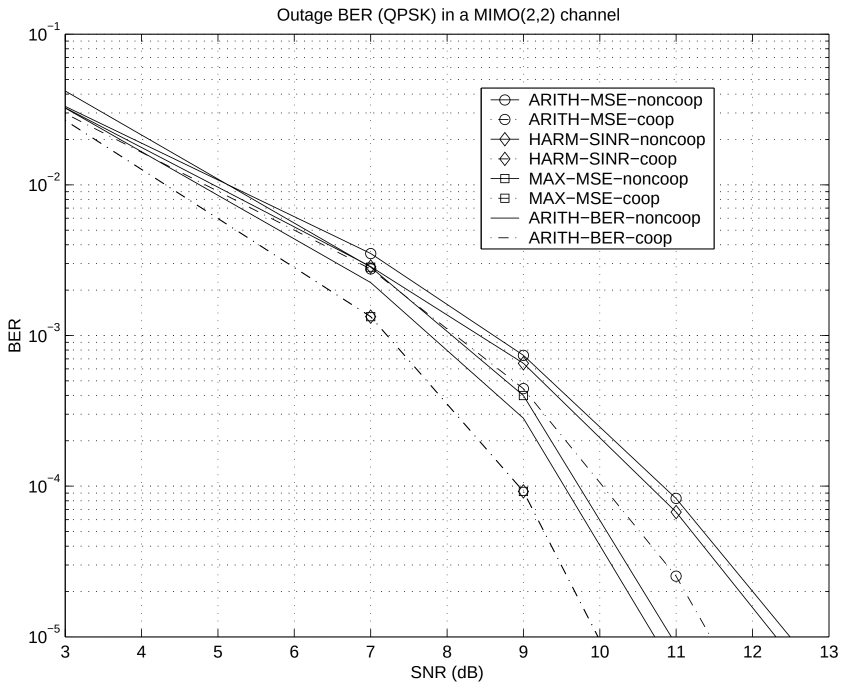


Figure 10: BER (at an outage probability of 5%) vs. SNR when using QPSK in a 2×2 MIMO channel with $L = 1$ for the ARITH-MSE, HARM-SINR, MAX-MSE, and ARITH-BER criteria with (coop) and without (noncoop) carrier cooperation.

It has been shown how to include additional constraints on the design to control the dynamic range of the transmitted signal and the PAR. We have also considered the more general scheme in which cooperation among different carriers is allowed to improve performance. We have obtained two multi-level water-filling practical solutions to optimize the spatial substreams at all carriers (one minimizes the maximum MSE and the other maximizes the harmonic mean of the SINR's) that perform very close to the optimal solution in terms of average BER with a low implementation complexity. Interestingly, for carrier-cooperative schemes, it turns out that exact optimal solution in terms of minimizing the average BER can be obtained in closed-form.

9 Acknowledgment

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Appendices

A Proof of Theorem 1

We first present a couple of lemmas and then proceed to the proof of Theorem 1.

Lemma 11 *[17, 9.H.1.h] If* $\mathbf{A}$ *and* $\mathbf{B}$ *are* $n \times n$ *positive semidefinite Hermitian matrices, then*

$$\mathrm{Tr}\,(\mathbf{AB}) \geq \sum_{i=1}^{n} \lambda_{A,i}\, \lambda_{B,n-i+1}$$

where $\lambda_{A,i}$ and $\lambda_{B,i}$ are the eigenvalues of $\mathbf{A}$ and $\mathbf{B}$ respectively in decreasing order.

Lemma 12 *Given a matrix $\mathbf{B} \in \mathbb{C}^{n_T \times L}$ and a positive semidefinite Hermitian matrix $\mathbf{R}_H \in \mathbb{C}^{n_T \times n_T}$ such that $\mathbf{B}^H \mathbf{R}_H \mathbf{B}$ is a diagonal matrix with diagonal elements in increasing order (possibly with some zero diagonal elements), it is always possible to find another matrix of the form $\tilde{\mathbf{B}} = \mathbf{U}_{H,1} \mathbf{\Sigma}_{B,1}$ of at most rank $\check{L} \triangleq \min(L, \mathrm{rank}(\mathbf{R}_H))$ that satisfies $\tilde{\mathbf{B}}^H \mathbf{R}_H \tilde{\mathbf{B}} = \mathbf{B}^H \mathbf{R}_H \mathbf{B}$ with $\mathrm{Tr}(\tilde{\mathbf{B}}\tilde{\mathbf{B}}^H) \leq \mathrm{Tr}(\mathbf{B}\mathbf{B}^H)$, where $\mathbf{U}_{H,1} \in \mathbb{C}^{n_T \times \check{L}}$ has as columns the eigenvectors of $\mathbf{R}_H$ corresponding to the $\check{L}$ largest eigenvalues in increasing order and $\mathbf{\Sigma}_{B,1} = \left[\mathbf{0}_{\check{L} \times (L - \check{L})} \; \mathrm{diag}(\{\sigma_{B,i}\})_{\check{L} \times \check{L}} \right] \in \mathbb{C}^{\check{L} \times L}$ has zero elements except along the rightmost main diagonal (which can be assumed real w.l.o.g.).*

Proof. Although the basic idea follows easily from the application of Lemma 11, the formal proof for arbitrary values of n_T, L, and $\mathrm{rank}(\mathbf{R}_H)$ becomes notationally involved. Since $\mathbf{B}^H \mathbf{R}_H \mathbf{B} \in \mathbb{C}^{L \times L}$ is diagonal with diagonal elements in increasing order, we can write

$$\mathbf{B}^H \mathbf{R}_H \mathbf{B} = \tilde{\mathbf{D}} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{D} \end{bmatrix}$$

where $\mathbf{D}$ is a square diagonal matrix (with real diagonal elements) of dimension $\check{L} \triangleq \min(L, \mathrm{rank}(\mathbf{R}_H))$. Using the singular value decomposition (SVD) [31], we can then write $\mathbf{R}_H^{1/2} \mathbf{B} = \mathbf{Q}\mathbf{\Sigma}$ where $\mathbf{Q} \in \mathbb{C}^{n_T \times n_T}$ is a unitary matrix whose columns are the left singular vectors, the right singular vectors (eigenvectors of $\tilde{\mathbf{D}}$) are the canonical vectors, and matrix $\mathbf{\Sigma} \in \mathbb{C}^{n_T \times L}$ is composed of zero elements and contains $\mathbf{D}^{1/2}$ in its top-right block so that $\mathbf{\Sigma}^H \mathbf{\Sigma} = \tilde{\mathbf{D}}$. In particular, if $n_T \geq L$, then $\tilde{\mathbf{D}} = \mathbf{D}$ and $\mathbf{\Sigma} = \begin{bmatrix} \mathbf{D}^{1/2} \\ \mathbf{0} \end{bmatrix}$, and if $n_T < L$, then $\mathbf{\Sigma} = \left[\mathbf{0} \; \mathbf{D}^{1/2} \right]$ (note that the position of matrix $\mathbf{D}^{1/2}$ within $\mathbf{\Sigma}$ is different to the classical definition of the SVD [31] because the diagonal elements of $\mathbf{B}^H \mathbf{R}_H \mathbf{B}$ are assumed in increasing order). Assuming that matrix $\mathbf{R}_H$ is nonsingular with eigendecomposition given by $\mathbf{R}_H = \mathbf{U}_H \mathbf{D}_H \mathbf{U}_H^H$, we can write

$$\mathbf{B} = \mathbf{R}_H^{-1/2} \mathbf{Q}\mathbf{\Sigma} = \mathbf{U}_H \mathbf{D}_H^{-1/2} \mathbf{U}_H^H \mathbf{Q}\mathbf{\Sigma}. \tag{58}$$

In case that $\mathbf{R}_H$ is singular, clearly $\mathbf{B}$ must be orthogonal to the null space of $\mathbf{R}_H$, otherwise this component could be made zero without changing the value of $\mathbf{B}^H \mathbf{R}_H \mathbf{B}$ and decreasing $\mathrm{Tr}(\mathbf{B}\mathbf{B}^H)$. Knowing that $\mathbf{B}$ must be orthogonal to the null space of $\mathbf{R}_H$, expression (58) is still valid using the pseudo-inverse of $\mathbf{R}_H$ instead of the inverse.

The idea now is to find another matrix $\mathbf{B}$ by changing the unitary matrix $\mathbf{Q}$ in (58) with the lowest possible value of $\mathrm{Tr}(\mathbf{B}\mathbf{B}^H)$ (note that any matrix $\mathbf{B}$ obtained from (58) satisfies by definition the desired constraint $\mathbf{B}^H \mathbf{R}_H \mathbf{B} = \tilde{\mathbf{D}}$). Using Lemma 11, $\mathrm{Tr}(\mathbf{B}\mathbf{B}^H)$ can be lower-bounded as follows:

$$\begin{aligned} \mathrm{Tr}(\mathbf{B}\mathbf{B}^H) &= \mathrm{Tr}\left(\mathbf{\Sigma}\mathbf{\Sigma}^H \tilde{\mathbf{U}}^H \mathbf{D}_H^{-1} \tilde{\mathbf{U}} \right) \\ &\geq \sum_{i=1}^{\check{L}} d_i \, \lambda_{H,i}^{-1} \end{aligned}$$

where $\tilde{\mathbf{U}} \triangleq \mathbf{U}_H^H \mathbf{Q}$, d_i is the ith diagonal element of $\mathbf{D}$ in increasing order and $\{\lambda_{H,i}\}_{i=1}^{\check{L}}$ are the $\check{L}$ largest eigenvalues of $\mathbf{R}_H$ in increasing order. If the d_i's are different, the lower bound is achieved by matrix $\tilde{\mathbf{U}}$ being a permutation matrix. For subsets of equal d_i's, the corresponding subblock in $\tilde{\mathbf{U}}$ can be any rotation matrix. Therefore, if $\tilde{\mathbf{U}}$ is chosen as a permutation matrix $\mathbf{P}$ that selects the $\check{L}$ largest eigenvalues of $\mathbf{R}_H$ in the same ordering as the d_i's, the lower bound is achieved. From (58), we obtain that the optimal $\mathbf{B}$ (in the sense of minimizing the value of $\text{Tr}\left(\mathbf{B}\mathbf{B}^H\right)$) has at most rank $\check{L}$ and is of the form $\mathbf{B} = \mathbf{U}_H \mathbf{D}_H^{-1/2} \mathbf{P} \mathbf{\Sigma} = \mathbf{U}_{H,1} \mathbf{\Sigma}_{B,1}$ where $\mathbf{U}_{H,1} \in \mathbb{C}^{n_T \times \check{L}}$ has as columns the eigenvectors of $\mathbf{R}_H$ corresponding to the $\check{L}$ largest eigenvalues in increasing order and $\mathbf{\Sigma}_{B,1} = [\mathbf{0} \ \text{diag}\left(\{\sigma_{B,i}\}\right)] \in \mathbb{C}^{\check{L} \times L}$ has zero elements except along the rightmost main diagonal (which can be assumed real w.l.o.g.). ■

Proof of Theorem 1. The constrained optimization problem to be solved is

$$\begin{array}{ll} \min\limits_{\mathbf{B}} & f_0\left(\mathbf{d}(\mathbf{E}\left(\mathbf{B}\right))\right) \\ \text{s.t.} & \text{Tr}\left(\mathbf{B}\mathbf{B}^H\right) \leq P_T \end{array}$$

where $\mathbf{d}(\mathbf{E})$ is the vector of diagonal elements of the MSE matrix $\mathbf{E}\left(\mathbf{B}\right) = \left(\mathbf{I} + \mathbf{B}^H \mathbf{R}_H \mathbf{B}\right)^{-1}$. It is mathematically convenient to assume the diagonal elements of $\mathbf{E}\left(\mathbf{B}\right)$ in decreasing order, *i.e.*, $d_i(\mathbf{E}) \geq d_{i+1}(\mathbf{E})$. Interestingly, this is without loss of generality. In fact, most reasonable objective functions (in particular, all the objective functions considered in Section 5) have a fixed preferred ordering of the arguments, *i.e.*, the value of the function is minimized with a very specific ordering of the arguments. In such cases, it suffices to relabel the arguments so that the preferred ordering is decreasing. In a more general case, however, a function may not have a fixed preferred ordering since it may depend on the specific value of the arguments. In such a case, since we are interested in minimizing the objective function, we can define and use instead the function $\tilde{f}_0\left(\mathbf{x}\right) = \min_{\mathbf{P} \in \Pi} f_0\left(\mathbf{P}\mathbf{x}\right)$, where $\mathbf{P}$ is a permutation matrix and Π is the set of the $L!$ different permutation matrices. The original minimization of f_0 without the ordering constraint is equivalent to the minimization of $\tilde{f}_0$ with the ordering constraint. Therefore, we can always assume that the function to be minimized has been properly defined so that the ordering constraint can be included without loss of generality (c.f. Section 5).

If f_0 is Schur-concave, it follows by Lemma 1 that $f_0\left(\boldsymbol{\lambda}(\mathbf{E})\right) \leq f_0\left(\mathbf{d}(\mathbf{E})\right)$ where $\boldsymbol{\lambda}(\mathbf{E})$ is the vector of eigenvalues of $\mathbf{E}$ in decreasing order. The lower bound $f_0\left(\boldsymbol{\lambda}(\mathbf{E})\right)$ is achieved if matrix $\mathbf{E} = \left(\mathbf{I} + \mathbf{B}^H \mathbf{R}_H \mathbf{B}\right)^{-1}$ is diagonal with diagonal elements in decreasing order or, equivalently, if $\mathbf{B}^H \mathbf{R}_H \mathbf{B}$ is diagonal with diagonal elements in increasing order. Furthermore, for any given $\mathbf{B}$, one can always find a rotation matrix $\mathbf{Q}$ so that $\mathbf{Q}^H \left(\mathbf{B}^H \mathbf{R}_H \mathbf{B}\right) \mathbf{Q}$ becomes diagonal and use instead the transmit matrix $\tilde{\mathbf{B}} = \mathbf{B}\mathbf{Q}$ improving the performance (by this rotation the utilized power remains the same). This implies that for Schur-concave functions, there is an optimal $\mathbf{B}$ with a structure such that $\mathbf{B}^H \mathbf{R}_H \mathbf{B}$ is diagonal with diagonal elements in increasing order.

If f_0 is Schur-convex, the opposite happens. By Lemma 2, it follows that $f_0\left(\mathbf{d}(\mathbf{E})\right)$ is minimized when $\mathbf{E}$ has equal diagonal elements. Furthermore, for any given $\mathbf{B}$, one can always find a rotation matrix $\mathbf{Q}$ so that $\mathbf{Q}^H \left(\mathbf{I} + \mathbf{B}^H \mathbf{R}_H \mathbf{B}\right)^{-1} \mathbf{Q}$ has identical diagonal elements by Lemma 3 (the sum of diagonal elements of $\mathbf{E}$ remains the same regardless of $\mathbf{Q}$), and use instead the transmit matrix $\tilde{\mathbf{B}} = \mathbf{B}\mathbf{Q}$ improving the performance (the transmit power remains the same). Therefore, for an optimal $\mathbf{B}$ we have that $\left[\left(\mathbf{I} + \mathbf{B}^H \mathbf{R}_H \mathbf{B}\right)^{-1}\right]_{ii} = \frac{1}{L} \text{Tr}\left(\mathbf{I} + \mathbf{B}^H \mathbf{R}_H \mathbf{B}\right)^{-1}$. Interestingly, regardless of the specific function f_0, the optimal $\mathbf{B}$ can be found by first minimizing $\text{Tr}\left(\mathbf{I} + \mathbf{B}^H \mathbf{R}_H \mathbf{B}\right)^{-1}$ (without imposing the constraint that the diagonal elements be equal) and then including the rotation to make the diagonal elements identical. The rotation can be found using the algorithm given in [34, Section IV-A] or with any rotation matrix $\mathbf{Q}$ that satisfies $\left|\mathbf{Q}_{ik}\right| = \left|\mathbf{Q}_{il}\right|, \forall i, k, l$ such as the DFT matrix or the Hadamard matrix (when the dimensions are appropriate such as a power of two [33, p. 66]). Regarding the minimization of $\text{Tr}\left(\mathbf{I}+\mathbf{B}^H\mathbf{R}_H\mathbf{B}\right)^{-1}$, since it is a Schur-concave function of the diagonal elements ($f_0(\mathbf{d}) = \sum_{i=1}^{L} d_i$), the previous result can be applied to show that there is an optimal $\mathbf{B}$ (excluding for the moment the rotation) such that $\mathbf{B}^H\mathbf{R}_H\mathbf{B}$ is diagonal with diagonal elements in increasing order.

Given that $\mathbf{B}^H\mathbf{R}_H\mathbf{B}$ is diagonal, it follows from Lemma 12 that $\mathbf{B}$ has at most rank $\check{L} \triangleq \min(L, \text{rank}(\mathbf{R}_H))$ and can be written as $\mathbf{B} = \mathbf{U}_{H,1}\mathbf{\Sigma}_{B,1}$ where $\mathbf{U}_{H,1} \in \mathbb{C}^{n_T\times\check{L}}$ has as columns the eigenvectors of $\mathbf{R}_H$ corresponding to the $\check{L}$ largest eigenvalues in increasing order and $\mathbf{\Sigma}_{B,1} = [\mathbf{0}\ \text{diag}(\{\sigma_{B,i}\})] \in \mathbb{C}^{\check{L}\times L}$ has zero elements except along the rightmost main diagonal.

Hence, we can write an optimal $\mathbf{B}$ as

$$\mathbf{B} = \begin{cases} \mathbf{U}_{H,1}\mathbf{\Sigma}_{B,1} & \text{for } f_0 \text{ Schur-concave} \\ \mathbf{U}_{H,1}\mathbf{\Sigma}_{B,1}\mathbf{V}_B^H & \text{for } f_0 \text{ Schur-convex} \end{cases}$$

where $\mathbf{U}_{H,1}$ and $\mathbf{\Sigma}_{B,1}$ are defined as before and $\mathbf{V}_B \in \mathbb{C}^{L\times L}$ is the rotation to make the diagonal elements of $\left(\mathbf{I}+\mathbf{B}^H\mathbf{R}_H\mathbf{B}\right)^{-1}$ identical. ■

B Proof of Lemma 4 $\left(f_0(\mathbf{x}) = \sum_i (w_i\, x_i)\right)$

Given a set of x_i's in decreasing order $x_i \geq x_{i+1}$, the function $f_0(\mathbf{x}) = \sum_i (w_i\, x_i)$ is minimized with the weights in increasing order $w_i \leq w_{i+1}$. To show this assume for a moment that for $i < j$ ($x_i \geq x_j$) the weights are such that $w_i > w_j$. We now show that the term $(w_i\, x_i + w_j\, x_j)$ can be minimized by simply swapping the weights:

$$\begin{aligned} & w_i\ (x_i - x_j) \geq w_j\ (x_i - x_j) \\ \Longleftrightarrow\quad & w_i\, x_i + w_j\, x_j \geq w_i\, x_j + w_j\, x_i. \end{aligned}$$

To prove that f_0 is Schur-concave, define $\phi(\mathbf{x}) \triangleq -f_0(\mathbf{x}) = \sum_i g_i(x_i)$ where $g_i(x) = -w_i\, x$. Function ϕ is Schur-convex because $g_i'(a) \geq g_{i+1}'(b)$ whenever $a \geq b$ [17, 3.H.2]. Therefore, f_0 is Schur-concave (see Definition 3). ■

C Proof of Lemma 5 $\left(f_0(\mathbf{x}) = \prod_i x_i^{w_i}\right)$

Given a set of strictly positive x_i's in decreasing order $x_i \geq x_{i+1} > 0$, the function $f_0(\mathbf{x}) = \prod_i x_i^{w_i}$ is minimized with the weights in increasing order $w_i \leq w_{i+1}$. To show this assume for a moment that for $i < j$ ($x_i \geq x_j$) the weights are such that $w_i > w_j$. We now show that the term $(x_i^{w_i}\, x_j^{w_j})$ can be minimized by simply swapping the weights:

$$\begin{aligned} & w_i \log\left(\frac{x_i}{x_j}\right) \geq w_j \log\left(\frac{x_i}{x_j}\right) \\ \Longleftrightarrow\quad & \left(\frac{x_i}{x_j}\right)^{w_i} \geq \left(\frac{x_i}{x_j}\right)^{w_j} \\ \Longleftrightarrow\quad & x_i^{w_i}\, x_j^{w_j} \geq x_i^{w_j}\, x_j^{w_i}. \end{aligned}$$

To prove that f_0 is Schur-concave, define $\phi(\mathbf{x}) \triangleq -\log f_0(\mathbf{x}) = \sum_i g_i(x_i)$ where $g_i(x) = -w_i \log x$. Function ϕ is Schur-convex because $g_i'(a) \geq g_{i+1}'(b)$ whenever $a \geq b$ [17, 3.H.2]. Since $f_0(\mathbf{x}) = e^{-\phi(\mathbf{x})}$ and function e^{-x} is decreasing in x, f_0 is Schur-concave [17, 3.B.1]. ■

D Proof of Lemma 6 ($f_0(\mathbf{x}) = \max_i \{x_i\}$)

From Definition 1, it follows that $f_0(\mathbf{x}) = \max_i \{x_i\} = x_{[1]}$. If $\mathbf{x} \prec \mathbf{y}$ it must be that $x_{[1]} \le y_{[1]}$ (from Definition 2) and, therefore, $f_0(\mathbf{x}) \le f_0(\mathbf{y})$. This means that f_0 is Schur-convex by Definition 3. ■

E Proof of Lemma 7 ($f_0(\mathbf{x}) = -\sum_i \left(w_i \left(x_i^{-1} - 1\right)\right)$)

Since the x_i's are strictly positive and in decreasing order $x_i \ge x_{i+1} > 0$, the function $f_0(\mathbf{x}) = -\sum_i \left(w_i \left(x_i^{-1} - 1\right)\right)$ is minimized with the weights in increasing order $w_i \le w_{i+1}$ (this can be similarly proved as was done in the proof of Lemma 4).

To show that f_0 is Schur-concave, define $\phi(\mathbf{x}) \triangleq -f_0(\mathbf{x}) = \sum_i g_i(x_i)$ where $g_i(x) = w_i \left(x^{-1} - 1\right)$. Since $g_i'(a) \ge g_{i+1}'(b)$ whenever $a \ge b$, it follows that ϕ is Schur-convex [17, 3.H.2] and, therefore, f_0 is Schur-concave (see Definition 3). ■

F Proof of Lemma 8 ($f_0(\mathbf{x}) = -\prod_i \left(x_i^{-1} - 1\right)^{w_i}$)

Given a set of strictly positive x_i's in decreasing order $x_i \ge x_{i+1} > 0$, the function $f_0(\mathbf{x}) = -\prod_i \left(x_i^{-1} - 1\right)^{w_i}$ is minimized with the weights in increasing order $w_i \le w_{i+1}$ (this can be similarly proved as was done in the proof of Lemma 4).

To show that f_0 is Schur-concave provided that $x_i \le 0.5$, define $\phi(\mathbf{x}) \triangleq \log(-f_0(\mathbf{x})) = \sum_i g_i(x_i)$ where $g_i(x) = w_i \log\left(x^{-1} - 1\right)$. Function ϕ is Schur-convex because $g_i'(a) \ge g_{i+1}'(b)$ whenever $0.5 \ge a \ge b$ [16] [17, 3.H.2]. Since $f_0(\mathbf{x}) = -e^{\phi(\mathbf{x})}$ and function $-e^x$ is decreasing in x, f_0 is Schur-concave [17, 3.B.1]. ■

G Proof of Lemma 9 ($f_0(\mathbf{x}) = \sum_i \frac{x_i}{1-x_i}$)

To prove that the function $f_0(\mathbf{x}) = \sum_i \frac{x_i}{1-x_i}$ is Schur-convex, rewrite it as $f_0(\mathbf{x}) = \sum_i g(x_i)$ where $g(x) = \frac{x}{1-x}$. Since g is convex, it follows that f_0 is Schur-convex [17, 3.H.2]. ■

H Analysis of the Convexity of the BER

In this appendix, we show that the BER and also the corresponding Chernoff upper bound are convex decreasing functions of the SINR and convex increasing functions of the MSE (for sufficiently small values of the MSE).

BER as a function of the SINR

To prove that the BER is convex decreasing in the SINR, it suffices to show that the first and second derivatives of $\mathcal{Q}\left(\sqrt{\beta\, x}\right)$ are negative and positive respectively (note that a positive scaling factor preserves

[16]Function $(1 - x)\, x$ is increasing in x for $0 \le x \le 0.5$.

monotonicity and convexity):

$$\frac{\partial \mathcal{Q}\left(\sqrt{\beta\, x}\right)}{\partial x} = -\sqrt{\frac{\beta}{8\pi}} e^{-\beta x/2} x^{-1/2} < 0 \qquad 0 < x < \infty$$
$$\frac{\partial^2 \mathcal{Q}\left(\sqrt{\beta\, x}\right)}{\partial x^2} = \frac{1}{2}\sqrt{\frac{\beta}{8\pi}} e^{-\beta x/2} x^{-1/2} \left(\frac{1}{x} + \beta\right) > 0 \qquad 0 < x < \infty.$$

The same can be done for the Chernoff upper bound $e^{-\beta x/2}$:

$$\frac{\partial e^{-\beta x/2}}{\partial x} = -\frac{\beta}{2} e^{-\beta x/2} < 0 \qquad 0 < x < \infty$$
$$\frac{\partial^2 e^{-\beta x/2}}{\partial x^2} = \left(\frac{\beta}{2}\right)^2 e^{-\beta x/2} > 0 \qquad 0 < x < \infty.$$

BER as a function of the MSE

To prove that the BER is convex increasing in the MSE, it suffices to show that the first and second derivatives are both positive:

$$\frac{\partial \mathcal{Q}\left(\sqrt{\beta\,\left(x^{-1}-1\right)}\right)}{\partial x} = \sqrt{\frac{\beta}{8\pi}} e^{-\beta\left(x^{-1}-1\right)/2} \left(x^3 - x^4\right)^{-1/2} \geq 0 \qquad 0 < x \leq 1$$
$$\frac{\partial^2 \mathcal{Q}\left(\sqrt{\beta\,\left(x^{-1}-1\right)}\right)}{\partial x^2} = \frac{1}{2}\sqrt{\frac{\beta}{8\pi}} e^{-\beta\left(x^{-1}-1\right)/2} \left(x^3 - x^4\right)^{-1/2} \left(\frac{\beta}{x^2} - \frac{3-4x}{x-x^2}\right) \geq 0 \qquad 0 < x \leq x_{z_1}, \quad x_{z_2} \leq x \leq 1.$$

where the zeros are $x_{z_1} = \frac{(\beta+3)-\sqrt{\beta^2-10\beta+9}}{8}$ and $x_{z_2} = \frac{(\beta+3)+\sqrt{\beta^2-10\beta+9}}{8}$ (it has been tacitly assumed that $\beta \leq 1$). It is remarkable that for $\beta = 1$ both zeros coincide, which means that the BER function is convex for the whole range of MSE values. To be more specific, BPSK and QPSK constellations satisfy this condition and, consequently, their corresponding BER function is always convex in the MSE.

The same can be done for the Chernoff upper bound $e^{-\beta\left(x^{-1}-1\right)/2}$:

$$\frac{\partial e^{-\beta\left(x^{-1}+1\right)/2}}{\partial x} = \frac{\beta}{2} e^{\beta/2} e^{-\beta x^{-1}/2} x^{-2} > 0 \qquad 0 < x < \infty$$
$$\frac{\partial^2 e^{-\beta\left(x^{-1}+1\right)/2}}{\partial x^2} = \frac{\beta}{2} e^{\beta/2} e^{-\beta x^{-1}/2} x^{-4} \left(\frac{\beta}{2} - 2x\right) \geq 0 \qquad 0 < x \leq \frac{\beta}{4}.$$

Therefore, the Chernoff upper bound is convex increasing in the MSE for MSE $\leq \beta/4$.

Concluding, as a rule-of-thumb, the BER and the Chernoff upper bound are convex increasing in the MSE for a BER $\leq 2 \times 10^{-2}$ (see [36] for more details).

I Proof of Lemma 10 ($f_0(\mathbf{x}) = \sum_i \text{BER}\left(x_i^{-1} - 1\right)$)

To prove that the function $f_0(\mathbf{x}) = \sum_i \text{BER}\left(x_i^{-1} - 1\right)$ is Schur-convex for $\theta \geq x_i > 0$ (for sufficiently small θ such that $\text{BER}\left(x_i^{-1} - 1\right) \leq 10^{-2}\ \forall i$), write $f_0(\mathbf{x}) = \sum_i g(x_i)$ where $g(x) = \text{BER}\left(x^{-1} - 1\right)$. Since function g is convex within the range $(0, \theta]$ (see Appendix H), it follows that f_0 is Schur-convex [17, 3.H.2]. ■

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