1 Use of Diversity

1.1 Receiver-Beamforming

For the scheme depicted in Fig. (1), a *QPSK* signal is transmitted such that the alphabet is composed by the following symbols $\mathbf{s}_i = \pm A\frac{\sqrt{2}}{2} \pm jA\frac{\sqrt{2}}{2}$ which are equally likely and statistically independent. The receiver front-end is composed by two antennae. For a given transmitted symbol $\mathbf{s}_i$, the signal model of the relevant information at the receiver matched filter outputs is given by:

$$\begin{aligned} \mathbf{r}_0 &= \mathrm{h}_o\mathbf{s}_i + \mathbf{n}_0 \\ \mathbf{r}_1 &= \mathrm{h}_1\mathbf{s}_i + \mathbf{n}_1 \end{aligned} \tag{1}$$

where h_0 and h_1 are the channel multiplicative terms (*frequency flat-fading*), $\mathbf{n}_0$ and $\mathbf{n}_1$ are the two relevant complex Gaussian noise terms, which are zero-mean, with mean-power N_0 Watts and statistically independent.

1. Provide the analytical expression for the decision rule under a *MAP criterion.* The probability density function for a Gaussian complex random variable is given by:

$$f_x(x) = \frac{1}{\pi\sigma_x^2} exp\left(-\frac{|x|^2}{\sigma_x^2}\right)$$

2. The two antenna outputs are linearly combined at the receiver as follows: $\widetilde{\mathbf{s}} = \lambda_0\mathbf{r}_0 + \lambda_1\mathbf{r}_1$. From the Cauchy-Schwartz inequality, provide the values for λ_0 and λ_1 that maximize the *signal-to-noise ratio*, that is, maximizing the following ratio (*Maximum-Ratio Combining criterion or MRC)*:

$$\lambda_0; \lambda_1 = arg\,max \frac{E\left[|\lambda_0 h_0\mathbf{s}_i + \lambda_1 h_1\mathbf{s}_i|^2\right]}{E\left[|\lambda_0\mathbf{n}_0 + \lambda_1\mathbf{n}_1|^2\right]} \tag{2}$$

3. Give the relationship between the Maximum-Ratio Combining solution obtained in (2.) and the *maximum channel capacity* solution for the problem.
4. Provide the analysis of the *Bit-Error Rate (BER)* at the *MRC* output.
5. Evaluate the channel capacity for the resulting end-to-end system as a function of the transmitted power A^2.

1.2 Transmitter-Beamforming

For the same *QPSK* constellation, we now consider that the transmitter is composed by two antennae and the receiver by a single antenna such that the information transmitted by the two antennae is given by:

$$\begin{aligned} \mathbf{s}_0^T &= \alpha_o\mathbf{s}_i \\ \mathbf{s}_1^T &= \alpha_1\mathbf{s}_i \end{aligned} \tag{3}$$

Due to the two channel contributions, the received signal is of the following form: $\widetilde{\mathbf{s}} = h_0\mathbf{s}_0^T + h_1\mathbf{s}_1^T + \mathbf{n}$ where $\mathbf{n}$ is the relevant complex Gaussian noise term, which is zero-mean, with mean-power N_0 Watts and $\{h_o, h_1\}$are the channel gains.

6. Give the expression of the total transmitted power.
7. Provide the optimal transmitter gains $\{\alpha_o, \alpha_1\}$ under a MAP/ML decision criteria at the receiver output with a total transmitted power constraint.
8. Verify the need of the knowledge of the *Channel-State Information* (*CSI*), that is, of $\{h_o, h_1\}$ by the transmitter for solution in (7.).
9. Evaluate the channel capacity for the resulting end-to-end system as a function of the transmitted power. Compare the result with the one in (5.).

1.3 Alamouti's Scheme

The communication system is now modified according to Fig. (2). Under the same transmission conditions, we have two transmitting elements and a single receiving antenna. Each antenna is transmitting a different *QPSK* symbol that they interchange in the next channel-use. Thus, after the reception of two consecutive symbols, the signal model becomes:

$$\begin{aligned} \mathbf{r}_0 &= \mathbf{r}(\mathrm{kT}) = \mathrm{h}_o \mathbf{s}_o + \mathrm{h}_1 \mathbf{s}_1 + \mathbf{n}_0 \\ \mathbf{r}_1 &= \mathbf{r}((\mathrm{k}+1)\,\mathrm{T}) = -\mathrm{h}_o \mathbf{s}^*{}_1 + \mathrm{h}_1 \mathbf{s}^*{}_0 + \mathbf{n}_1 \end{aligned} \tag{4}$$

10. Show that Eq. (4) admits the following representation and show that $\mathbf{M}^H\mathbf{M}$ is a diagonal matrix:

$$\begin{bmatrix} \mathbf{r}_0 \\ \mathbf{r}_1^* \end{bmatrix} = \mathbf{M} \begin{bmatrix} \mathbf{s}_0 \\ \mathbf{s}_1 \end{bmatrix} + \begin{bmatrix} \mathbf{n}_0 \\ \mathbf{n}_1^* \end{bmatrix} \tag{5}$$

11. Provide the equations for an optimal detection of symbols $\mathbf{s}_0$ and $\mathbf{s}_1$ under a *MAP* criterion.
12. Provide the analysis of the *BER* for the system developed in the previous question. Compare the results with the *BER* obtained for the *MRC* of the system given in Fig. (1).
14. Provide an expression for the channel capacity associated to the Alamouti's scheme as a function of the transmitted power.
15. Give an assessment about the degree of optimality of Alamouti's code with special emphasis on the use of the transmitted power in the achievable channel capacity.
13. Provide a detailed description of the advantages and drawbacks associated to the three analyzed systems.

2 Fast-Fading Channels

We consider the following *frequency-flat fading* channel model:

$$y\,[n] = h\,[n]\,x\,[n] + w\,[n]$$

where $h\,[n]$ is the random channel gain, $x\,[n]$ is the transmitted sequence and $w\,[n]$ is the additive, Gaussian noise, such that, $E\left[|h\,[n]|^2\right] = 1$, $E\left[|x\,[n]|^2\right] = S_T$ and $E\left[|w\,[n]|^2\right] = N_o$. All magnitudes are zero-mean and complex.

1. Consider that the channel is *stationary* (*time-invariant*) such that $h\,[n] = 1$. Give the expression of the channel capacity that we will denote C_{AWGN} all along the problem.
2. We now consider the channel gain as a random variable. Provide an interpretation of the following magnitude:

$$C_{fast} = E\left[log\left(1 + |h|^2 \frac{S_T}{N_o}\right)\right]$$

where $E\,[.]$ is the expected value with respect to the channel gain $|h|^2$ *probability density function*.
3. Show that (Jensen's inequality):

$$C_{fast} \leq C_{AWGN}$$

4. Show that in the *low-SNR regime*:

$$C_{fast} \simeq C_{AWGN}$$

5. Give an expression for C_{fast} in the *high-SNR regime*. Provide an expression of the penalty of C_{fast} with respect C_{AWGN} in such a regime. Observe and justify the influence of the term $E\left[|h\,[n]|^2\right]$ in the penalty.

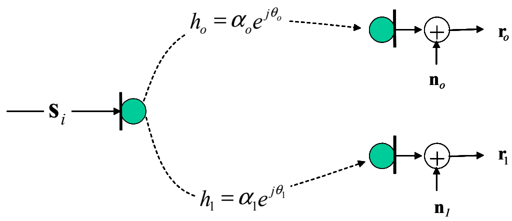

Figure 1:

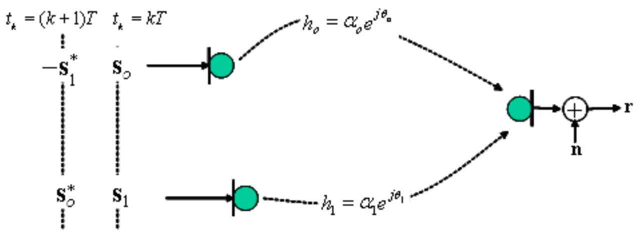

Figure 2: