A NOVEL ESTIMATOR AND PERFORMANCE BOUND FOR TIME PROPAGATION AND DOPPLER BASED RADIO-LOCATION.

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ABSTRACT

This paper presents the theoretical accuracy limits of the geolocation algorithms based on TOAs measurements exploiting the fact that the mobile is moving in a known or unknown direction. The developed expresions show us the possible improvements in terms of accuracy and/or availability due to the diversity created with the movement. A simple algorithm based on the use of TOAs drift estimation is also presented in order to compare its performance with the developed theoretical limits. This proposed estimator attains the theoretial limits under certain conditions.

1. INTRODUCTION

A problem of growing importance in mobile communication networks is estimating the position of mobile terminals. One of the most common approach is getting a position estimation by combining several time of arrival (TOA) or time difference of arrival (TDOA) measurements amongst a set of references which are normally base stations and/or satellites [5],[1]. The major problems in this approach are the non-linear relationship between measurements (TOA or TDOA) and position parameters (Cartesian coordinates) and the limited availability of base stations, specially in rural environments.

In static scenarios, where the mobile is not moving, there is a minimum number of references needed to compute a valid position and the dilution of precision (DOP) is defined by the geometry of the mobile and reference positions. In dynamic scenarios, the movement of the mobile generates a diversity that can be exploited. This diversity can both reduce the minimum number or references and/or improve the accuracy of the algorithm. In fact, the DOP is also modified by the presence of movement. Concretely, a complete knowledge of the mobile speed improves the accuracy of the position estimate and the availability of the algorithm. In the recent literature, several articles have presented the usage of the satellite speed knowledge to improve the position accuracy and/or availability [2]. This approach is similar to the one presented in this paper.

This paper presents the Cramer Rao Lower Bound (CRLB) of the accuracy in location applications based on TOA measurements when the mobile is moving in a certain direction. Later, for space reasons we develop the expression of the CRLB of the position estimation in the specific case of a complete knowledge of the speed. This approach can be applied to the case of the location algorithm assisted with an inertial sensor system that can supply the speed of the mobile and its direction (compass is required). Finally, we show a simple efficient location algorithm based on the TOA and drift measurements that can attain the developed CRLB under certain conditions.

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2. CRLB OF THE POSITION ACCURACY

In this section it will be develop the CRLB of the position estimation using as information the TOA measurements coming from L references (base stations) in the case that the mobile is moving in a certain direction. First, let us assume that a finite length window (K) is used to observe the TOA measurements evolution delivered at a rate r (samples per second). The available measurements of the l-th reference can be expressed as follows:

$$\mathbf{toa}_l = \begin{bmatrix} toa_{l,1} & \cdots & toa_{l,K} \end{bmatrix}^T + \mathbf{n}_l \qquad (1)$$

where $toa_{l,n}$ represents the TOA measurement of the l-th reference at the n-th instant of time, $\mathbf{n}_l$ is the noise added vector and T denotes the transpose operation. For simplicity reasons, it has been considered that TOA measurements are provided in meters, this is all time measurements are transformed into distance measurements. From this information, we can estimate the TOA in the middle of the used window $\left(\widehat{t}_l\right)$ and its associated drift $\left(\widehat{d}_l\right)$ with a certain model. Logically the simplest estimation is performed with the weighted least square (WLS) algorithm and can be performed as follows:

$$\begin{bmatrix} \widehat{t}_l \\ \widehat{d}_l \end{bmatrix} = \left(\mathbf{A}^H \mathbf{R}_l^{-1} \mathbf{A}\right)^{-1} \mathbf{A} \mathbf{R}_l^{-1} \cdot \mathbf{toa}_l \qquad (2)$$

where the $\mathbf{R}_l = E\left[\mathbf{n}_l \cdot \mathbf{n}_l^T\right]$ and matrix A is given by:

$$\mathbf{A} = \begin{bmatrix} 1 & \cdots & 1 \\ -(K-1)/2 & \cdots & (K-1)/2 \end{bmatrix}^T \quad (3)$$

where a constant-drift model has been assumed. Logically, this constant-drift model may not to fit with the real TOA measurements evolution, so a higher order can be used achieving a higher variance in the estimation of $\widehat{t_l}$ and $\widehat{d_l}$ but avoiding any bias in the estimation [3].

Once the mean TOA $(\widehat{t_l})$ and the associated drift $(\widehat{d_l})$ of the L references have been estimated without bias, the non-linear relationship between these $2L$ parameters and the parameters of interest in the location problem (position and speed) can be expressed as follows:

$$\mathbf{p} = \begin{bmatrix} \widehat{t}_1 \\ \vdots \\ \widehat{t}_L \\ \widehat{d}_1 \\ \vdots \\ \widehat{d}_L \end{bmatrix} = \begin{bmatrix} f(\mathbf{x}, \mathbf{x}_1) \\ \vdots \\ f(\mathbf{x}, \mathbf{x}_L) \\ g(\mathbf{x}, \mathbf{s}, x_1) \\ \vdots \\ g(\mathbf{x}, \mathbf{s}, x_L) \end{bmatrix} + \mathbf{n}_{td} = \begin{bmatrix} \mathbf{F}(\mathbf{x}, \mathbf{x}_1^L) \\ \mathbf{G}(\mathbf{x}, \mathbf{s}, \mathbf{x}_1^L) \end{bmatrix} + \mathbf{n}_{td} \quad (4)$$

where $\mathbf{x}$ is the mobile position, $\mathbf{x}_l$ is the position of the l-th reference ($\mathbf{x}_1^L$ denotes the set of L reference positions), $\mathbf{n}_{td}$ is the noise estimation vector in (2) and the scalar functions f and g can be expressed as:

$$f(\mathbf{x}, \mathbf{x}_l) = |\mathbf{r}_{x,x_l}| \quad (5)$$

$$g(\mathbf{x}, \mathbf{s}, \mathbf{x}_l) = \left|\frac{\mathbf{s}}{r}\right| \cdot \cos\left(\measuredangle \mathbf{s} - \measuredangle \mathbf{r}_{x,x_l}\right) \quad (6)$$

where $\mathbf{r}_{x,x_l}$ is the vector from the mobile position to the l-th reference position, $\mathbf{s}$ indicates the speed of the mobile, r denotes the rate at witch the original TOA measurements are delivered, $|\ |$ denotes the module operation and $\measuredangle$ indicates the angle of a vector in any Cartesian system included in the plane defined by the vectors $\mathbf{s}$ and $\mathbf{r}_{x,x_l}$. In order to find the CRLB, we have to linearize (4) as follows:

$$\mathbf{p} \approx \begin{bmatrix} \mathbf{F}(\mathbf{x}_0, \mathbf{x}_{1-L}) \\ \mathbf{G}(\mathbf{x}_0, \mathbf{s}_0, \mathbf{x}_{1-L}) \end{bmatrix} + \begin{bmatrix} \frac{\partial \mathbf{F}}{\partial \mathbf{x}^T} & 0 \\ \frac{\partial \mathbf{G}}{\partial \mathbf{x}^T} & \frac{\partial \mathbf{G}}{\partial \mathbf{s}^T} \end{bmatrix} \mathbf{e} + \mathbf{n}_{td} \quad (7)$$

where x_0 and s_0 are the true values of the position and speed to be estimated and $\mathbf{e} = \left[(\mathbf{x}-\mathbf{x}_0)^T \ (\mathbf{s}-\mathbf{s}_0)^T\right]^T$ is the error vector formed with the position and speed error. In order to simplify the notation, we can rewrite the previous expression as follows:

$$\begin{aligned} \mathbf{p} &\approx \begin{bmatrix} \mathbf{F}(\mathbf{x}_0) \\ \mathbf{G}(\mathbf{x}_0, \mathbf{s}_0) \end{bmatrix} + \begin{bmatrix} \mathbf{C}_{FX}^T & \mathbf{0} \\ \mathbf{C}_{GX}^T & \mathbf{C}_{GS}^T \end{bmatrix} \mathbf{e} + \mathbf{n}_{td} \quad (8) \\ \mathbf{p} &\approx \mathbf{p}(\mathbf{x}_0, \mathbf{s}_0) + \mathbf{C}^T \mathbf{e} + \mathbf{N} \end{aligned}$$

where the matrices $\mathbf{C}_{FX}^T$, $\mathbf{C}_{GX}^T$ and $\mathbf{C}_{GS}^T$ represent the derivatives shown in (7). Finally, the CRLB for the joint position and speed estimation can be expressed as follows:[4]

$$\mathbf{R}_\epsilon = \left(\mathbf{C}\mathbf{R}_{td}^{-1}\mathbf{C}^H\right)^{-1} \quad (9)$$

where $\mathbf{R}_{td}$ is the covariance matrix of the estimation of the vector $\mathbf{p}$, this is the variance of the elements of vector $\mathbf{n}_{td}$, and can be expressed as it follows:

$$\begin{aligned} \mathbf{R}_{td} &= diag\left(\mathbf{R}_t, \mathbf{R}_d\right) \quad (10) \\ \mathbf{R}_t &= diag\left(\sigma_{t_1}^2, ..., \sigma_{t_L}^2\right) \\ \mathbf{R}_d &= diag\left(\sigma_{d_1}^2, ..., \sigma_{d_L}^2\right) \quad (11) \end{aligned}$$

where $\sigma_{t_l}^2$ is the variance of the estimation of the l-th TOA and $\sigma_{d_l}^2$ is the variance of the l-th TOA drift in (2). Following the generic expression of the CRLB shown in (9) and the definition of the covariance matrices and linearization matrices defined in (10) and (8), it is not difficult to show that:

$$\mathbf{R}_\epsilon = \begin{pmatrix} \mathbf{C}_{FX}\mathbf{R}_t^{-1}\mathbf{C}_{FX}^T + \mathbf{C}_{GX}\mathbf{R}_d^{-1}\mathbf{C}_{GX}^T & \mathbf{C}_{GX}\mathbf{R}_d^{-1}\mathbf{C}_{GS}^T \\ \mathbf{C}_{GS}\mathbf{R}_d^{-1}\mathbf{C}_{GX}^T & \mathbf{C}_{GS}\mathbf{R}_d^{-1}\mathbf{C}_{GS}^T \end{pmatrix}^{-1} \quad (12)$$

This last expression is, in fact the Fisher information matrix of the joint estimation of the speed and position. Here, it is opened a new variant of problems where a partial knowledge of speed can exploit the diversity created by the mobile. Concretely, in this paper we will develop the CRLB of the position estimation under a complete knowledge of the speed. In this case, only the first element of the previous matrix has to be inverted. So, the final CRLB for position estimation is:

$$\mathbf{R}_x = \left(\mathbf{C}_{FX}\mathbf{R}_t^{-1}\mathbf{C}_{FX}^T + \mathbf{C}_{GX}\mathbf{R}_d^{-1}\mathbf{C}_{GX}^T\right)^{-1} = \left(\mathbf{R}_{x_T}^{-1} + \mathbf{R}_{x_D}^{-1}\right)^{-1} \quad (13)$$

This last expression has been divided into two terms: The classical covariance matrix produced by the use of the estimated TOAs $\left(\mathbf{R}_{x_T}^{-1}\right)$ and a new term that shows the contribution of the use of the estimated TOA-drifts $\left(\mathbf{R}_{x_D}^{-1}\right)$. Now we can develop the expressions of these two terms in a 2-Dimensional location problem assuming that the variance of the TOA measurement of each reference is constant along the observation window. To develop this expression, we only have to take the definition of f and g functions in (5),(6) and the expression of linearization matrices of (7) and (8).

$$\mathbf{R}_{x_T}^{-1} = \sum_{l=1}^{L} K\sigma_l^{-2} \cdot \overline{\mathbf{r}}_{x,x_l}\overline{\mathbf{r}}_{x,x_l}^T \quad (14)$$

$$\mathbf{R}_{x_D}^{-1} = \sum_{l=1}^{L} \frac{K^3\sigma_l^{-2}}{12} \cdot \frac{|\mathbf{s}|^2/r^2}{|\mathbf{r}_{x,x_l}|^2} \cdot \sin^2\left(\measuredangle \mathbf{s} - \measuredangle \mathbf{r}_{x,x_l}\right) \cdot \overline{\mathbf{r}}_{x,x_l}^{\top}\overline{\mathbf{r}}_{x,x_l}^{\top T} \quad (15)$$

where $\overline{\mathbf{r}}_{x,x_l}$ is the unitary vector in the direction between mobile position and l-th base station position and $\top$ denotes the perpendicular vector. Note that the rate of the TOA measurements depends on the coherent distance and the speed of the mobile. In fact, the maximum value for the rate is limited by the time spent by the mobile to cover the coherence distance $(r_{max} = (d_{coh}/|s|)^{-1})$.

Equations (14) and (15) show that the effect of TOA estimations ($\mathbf{R}_{x_T}^{-1}$) does not depend on the distance of the mobile but uniquely depends on the geometry of the problem, this is the different position of the references. Contrary, the contribution of the TOA-drift estimates ($\mathbf{R}_{x_D}^{-1}$) depends on the relationship between the speed and the distances between the references and the mobile. This behavior is due to the fact that the sensitivity of drift changes is inversely proportional to the distance between the reference and mobile. Note also, there is no contribution of the drift estimates if the mobile is not moving or if the mobile is moving in the line that links the mobile and the reference. This last behavior is due to the fact that there is no difference in the TOA-drift measurement in all the points of that line. Additionally it can be seen the different evolution of both terms in function of the window size. The cubic decreasing evolution of the term $\mathbf{R}_{x_D}^{-1}$ in front of the linear decreasing evolution of $\mathbf{R}_{x_T}^{-1}$ shows us that with a minimum size of the observation window, the TOA-drift related term will dominate in the estimation accuracy, so a nearly optimum algorithm based only on drift measurements would be devised.

Finally, one of the most common way to check the minimum number of references needed to compute a valid position is finding the number of references needed to produce a full-rank inverse covariance matrix ($\mathbf{R}_x^{-1}$)[6]. This assure us that the position estimation presents a finite variance in all possible directions. In the particular case of a 2-Dimensional location problem, the inverse covariance matrix is full-rank with the contribution of both terms (TOA and TOA-drift) of a unique base station. Note that the eigen-values of these two matrices are ortogonal following the definition shown in (14) and (15) so full-rank condition is assured. This means that if we can solve the ambiguities that we will present in the next section, the mobile can be located using uniquely one reference.

3. SIMPLE ESTIMATOR

In this section, we present a simple position estimator based on the TOA measurements that exploit the knowledge of the speed attaching the CRLB under certain conditions. If we revise the expression of the CRLB developed, we will realize that the two presented terms are related with the contribution of the two kind of measurements: TOA and TOA-drift. The estimator presented in this section compute the position of the mobile using the TOA estimate ($\widehat{t}_l$) and an estimation of the angle of the vector $\mathbf{r}_{x,x_l}$ based, uniquely, on the TOA-drift estimate ($\widehat{d}_l$). The relation between the angle of the vector $\overline{\mathbf{r}}_{x,x_l}$ and the TOA-drift is as follows:

$$\widehat{\measuredangle\overline{\mathbf{r}}_{x,x_l}} = \measuredangle\mathbf{s} \pm \arccos\left(\frac{\widehat{d}_l \cdot r}{|\mathbf{s}|}\right) \tag{16}$$

The ambiguity in the sign corresponds to the two trajectories that produce exactly the same TOA measurements evolution. This ambiguity has to be solved with a higher level algorithm that performs, as maximum, a 2^L search along all possible combination. Note that there are two possible trajectories per reference, so there are a total of 2^L possible combinations. Once, the angle of the vector $\overline{\mathbf{r}}_{x,x_l}$ has been estimated, the proposed location algorithm with a unique reference for the 2-Dimension case can be expressed as:

$$\widehat{x}_l = \widehat{t}_l \cdot \begin{bmatrix} \cos\left(\widehat{\measuredangle\overline{\mathbf{r}}_{\mathbf{x},x_l}}\right) \\ \sin\left(\widehat{\measuredangle\overline{\mathbf{r}}_{x,x_l}}\right) \end{bmatrix} \tag{17}$$

This estimator has the same variance as the CRLB if the model used in the WLS approach to estimate $\widehat{t}_l$ and $\widehat{d}_l$ does not produce bias. Once we have the estimation with a unique reference, we can extend the algorithm to a generic L reference algorithm only weighting the contributions of all used references with the individual covariance matrices as it follows:

$$\widehat{\mathbf{x}} = \left(\sum_{l=1}^{L} \mathbf{R}_{x_l}^{-1}\right)^{-1} \sum_{l=1}^{L} \mathbf{R}_{x_l}^{-1}\widehat{\mathbf{x}}_l \tag{18}$$

where $\mathbf{R}_{x_l}^{-1}$ is the inverse of the CRLB expressed in (13) for a unique reference.

The following section offers the performance analysis of the proposed estimator compared with the CRLB.

4. PERFORMANCE ANALYSIS

This section shows an example of the application of the proposed algorithm compared with the CRLB. The scenario consists in a unique base station and a mobile moving in a circular trajectory around the reference. This is a case where the linear model can be used to estimate the TOA and TOA-drift without bias, but we can extend the model with a higher order estimation process in order to avoid the bias in the estimates $\widehat{t}_l$ and $\widehat{d}_l$ in a generic trajectory.

Concretely we will simulate the case of a mobile terminal moving in a circular trajectories around a base station at a radius of 100 m at different speeds and with a TOA measurement rate of 3 $samples/sec$. As it has been explained before, the maximum rate is limited by the coherence space and the speed of the mobile, so this assumption of the rate is according to the coherence distance at a 900 Mhz radio link and at the minimum assumed speed (1 m/s). Figure (1) shows the variance in the individual estimation of the TOA in the middle point of the observation window and the TOA-drift as a function of the selected length for the window.

The variance of the original TOA has been assumed to be 40 m. In this picture, it can be seen the different evolution of the individual estimations. The TOA estimate tends to a linear decrease and the TOA-drift estimate tends to a cubic decrease. Now with theses estimates in mind, we can apply the algorithm shown in the previous section and compare its performance with the theoretical limits. The following picture illustrate the evolution of the mean square error in the position system, this is the trace of the covariance error matrix, as a function of the selected time window, for a different radius of the trajectory .

In figure (2), it can be seen the variance evolution of the position estimator as a function of the window size. It can be clear seen that with a minimum size of the window, the algorithm proposed achieve the CRLB. With smaller sizes

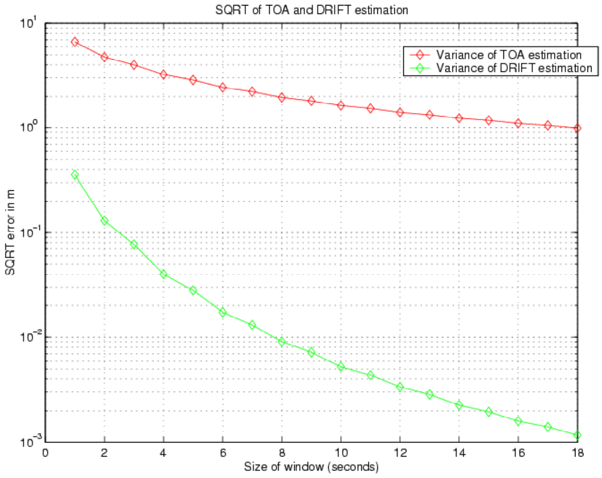

Figure 1: Variance in the individual estimation process of TOA and TOA-drift

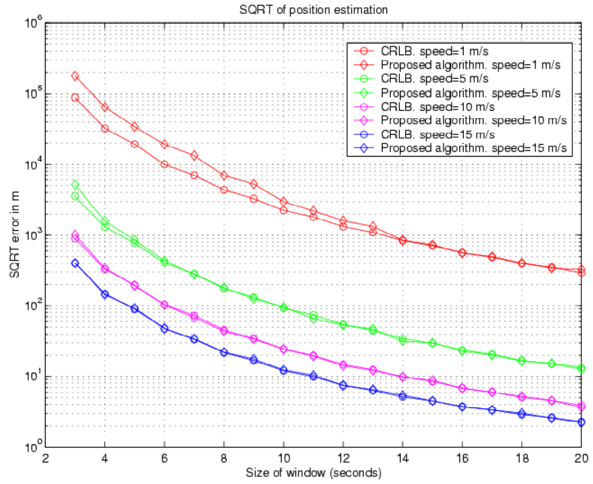

Figure 2: Mean square error in the position estimation

of the window, the variance in the angle estimation can not be considered Gaussian distributed after the function $\cos^{-1}$ in the expression (16), so the performance of the algorithm does not attain the CRLB. This effect is the classical effect in the phase estimation process when the SNR is not enough high.

It can also be seen that with a small value of the windows the CRLB tends to infinite due to the fact, that in this simulation only one reference is considered and then, if there is no contribution of the TOA-drift measurements the inverse of the covariance matrix (13) tend to be not full-rank, so the covariance matrix tend to be infinity (not availability). This situation is exactly the same that when no movement is performed by the mobile.

5. CONCLUSIONS

This paper develops the CRLB of the position accuracy in a dynamic scenario showing the possible diversity created by the own movement of the mobile. Concretely, it has been developed the CRLB in the case of a complete knowledge of the mobile and it has been shown the improvements in the accuracy and availability. It has been proven that the estimation of the TOA-drift with a linear or higher order model reduces the CRLB of the position estimation and generate a diversity that can be exploited to reduce the minim number of references (normally base stations) to perform a valid position. One of the easy way to understand this point is the following reflection. If the mobile is moving in a completely known direction, this scenario is exactly the same in witch a mobile is fixed and the references are moving and their positions are always known. This scenario generates diversity because the dilution of precision (DOP) matrix is different at each point and can perform a valid position (not infinite covariance matrix). This diversity collapses in the case of no moving because the DOP matrix is exactly the same in all points and then the diversity is not created.

The second part of this paper has presented a simple algorithm to exploit the knowledge of the speed and the estimated TOA-drift. This algorithm is based on the direct transformation of the TOA-drift into an estimation of the angle between the reference and the mobile position. A very simple trajectory simulation has shown the performance of the algorithm and the comparison with the CRLB. The results show that with a minimum size of the observation time window, the algorithm attaches the CRLB. It has been also shown the different evolution of the estimations coming from the use of the TOA estimates and from the TOA-drift estimates. Finally, this simulation also shows that the minimum number of references in a location algorithm based on TOA, this is two references, can be reduced with a complete knowledge of the speed to a single one.

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