A ROBUST MULTIPATH MITIGATION TECHNIQUE FOR TIME-OF-ARRIVAL ESTIMATION

Jaume Riba and Andreu Urruela

Department of Signal Theory and Communications. Universitat Politècnica de Catalunya.
Campus Nord, Ed. D5, Jordi Girona 1 i 3, 08034, Barcelona (Spain).
Email: {jriba,andreu}@gps.tsc.upc.es

ABSTRACT

The (frequency-selective) multipath propagation and the limited bandwidth available in GSM leads to high inacuracies of the conventional Time-Of-Arrival (TOA) estimation methods used by mobile location techniques. In this paper, a robust technique is proposed to ameliorate this phenomenon.

1. INTRODUCTION

The problem of Time-of-arrival (TOA) estimation of the Line-Of-Sight (LOS) propagation is becoming an important area of research. Mobile location constitutes an important application in which an estimation of the mobile position is delivered from several TOA estimates. For example, in the down-link case, TOA estimates can be computed from the signals broadcasted by Base Stations and/or Satellites. The present paper has been developed in the context of the IST project EMILY, which is focused on the coupling between terrestrial and satellite signals for positioning.

Multipah propagation is the major undesired effect to consider in the formulation of the TOA estimation problem. The technique developed in this paper, as well as other recently proposed techniques [1], works with the hypothesis that the LOS exits, but it is contaminated by multipath propagation (with TOAs higher than the LOS TOA) in an unknown manner. It is noted that the Non-Line-Of-Sight (NLOS) case should be treated in a different way, for example, by exploiting prior information and redundancy during the hybridization among all the computed TOAs [2]. This higher level data processing is not studied here.

In the presence of multipath, classical TOA estimation techniques lead to (mainly positive) bias in the resulting TOA estimate of the LOS. The worst-case bias is produced by a multipath dispersion of the same order of magnitude as the inverse signal bandwidth, which determines the natural resolution capability. High resolution techniques have proven useful to ameliorate this problem [1], but their use is limited to channels consisting of a linear combination of a finite number of time-delayed Dirac delta pulses. In practice, especially in urban scenarios, this kind of model is too much poor. Then, the natural formulation of the problem becomes the joint estimation of the TOA LOS and the multipath channel. In this contribution, we propose a simpler technique that tries to avoid channel estimation while still reducing the bias effect. To this end, we propose a new method that takes into account the statistical structure of the multipath, following similar ideas proposed by one of the authors in [3] and [4].

This work is partially supported by the European Commission under IST Project EMILY IST-2000-26040 and by the Spanish and Catalan goverments: TIC2001-2356-C02-01, TIC2000-1025, TIC99-0849, 2000SGR-00083, 2001SGR-00268.

2. STATISTICAL CHANNEL MODEL

The most simple channel model consists of a linear combination of M time-delayed Dirac delta pulses with amplitudes a_m and delays τ_m:

$$h_{ch}(t) = \sum_{m=1}^{M} a_m \delta(t - \tau_m) \qquad (1)$$

with $\tau_1 < \tau_2 < \ldots < \tau_M$. Therefore, at the receiver filter output of time response $h_r(t)$, and assuming a known data-burst (e.g., the midamble in a GSM normal burst), an estimation of the following signal can be performed:

$$r(t) = h_s(t) * h_{ch}(t) * h_r(t) = \sum_{m=1}^{M} a_m g(t - \tau_m) \qquad (2)$$

where

$$g(t) = h_s(t) * h_r(t) \qquad (3)$$

$r(t)$ can be estimated, for instance, by means of a cross-correlation-based technique using the known training sequence at the receiver.

The purpose of TOA estimation techniques for positioning is to estimate τ_1, which is associated to the LOS propagation. Of course, ignoring the presence of the multipath will lead to a significant bias in the estimated τ_1. The worst case corresponds to a multipath dispersion (approximately) lower than the inverse pulse bandwidth, (denoted as D_r and referred to as resolution time), and higher than the desired timing accuracy of the problem (denoted as D_a). Therefore, the multipath TOAs in the range $D_a < \tau_m - \tau_1 < D_r$ are the most susceptible of producing significant bias to the estimated τ_1.

The application of the maximum likelihood principle to the problem at hand requires the joint estimation of the complete set of delays $\{\tau_m\}$ and amplitudes $\{a_m\}$. One way to avoid this is to use high resolution techniques, as those proposed in [1]. However, both methods are not robust to the presence of model errors, i.e., errors on the model order estimation, M, and/or the presence of diffuse multipath.

To solve the previous limitations, we adopt a statistical model of the multipath, similar to that proposed in [5], which assumes that the time dispersion is produced by a single second path, but its associated amplitude and time dispersion are unknown:

$$h'_{ch}(t) = b\delta(t-\tau) + a\delta(t-\tau-D) \tag{4}$$

In the previous expression, b and a are the complex amplitudes of the first (LOS) and second path respectively and D is the (positive) time dispersion of the second path. The nuisance parameters are now endowed with priors which express our amount of knowledge depending on the scenario considered. Following some conclusions found in the literature [5], the prior of D (also called Power Delay Spectrum) will be modelled by a one-sided exponential decaying function of hyper-parameter σ_D^2, also called delay-spread. On the other hand, a will be modelled as a zero-mean random variable of variance σ_a^2, without assuming any specific prior for it. It is important to note that, although the model (4) includes only a single second path, endowing the path delay D with a certain prior will lead to a technique that will be also robust in the presence of diffuse multipath. This statement will be confirmed by simulations.

The main idea of our contribution is to view the second term of (4) as a noise term which is added to the estimation/measure noise, and whose color depends on the assumed priors. With the purpose of simplifying the mathematical description of the second order statistics of the multipath interference, we will use the frequency domain. From (4) we can write:

$$H'_{ch}(\omega) = e^{-j\omega\tau}\left(b + ae^{-j\omega D}\right) \tag{5}$$

and using it in (2) we can write:

$$R(\omega) = H_s(\omega)H_r(\omega)e^{-j\omega\tau}\left(b + ae^{-j\omega D}\right) \tag{6}$$

After sufficient sampling in the frequency domain at points ω_p, with $0 \le p \le P-1$, to neglect time aliasing, we can write the following discrete model:

$$\mathbf{r} = \mathbf{g} \odot \mathbf{e}_\tau \odot (b\mathbf{1} + \mathbf{u}_{a,D}) + \mathbf{w} \tag{7}$$

where $[\mathbf{r}]_p = R(\omega_p)$, $[\mathbf{g}]_p = H_s(\omega_p)H_r(\omega_p)$, $[\mathbf{e}_\tau]_p = e^{-j\omega_p\tau}$, $\mathbf{1}$ denotes the all-ones vector, and $[\mathbf{u}_{a,D}]_p = ae^{-j\omega_p D}$. Observe that, in the previous model, $\mathbf{g}$ and $\mathbf{u}_{a,D}$ constitute the known and unknown signal components, respectively. Finally an unknown white noise term vector $\mathbf{w}$ has been included. Its exact statistical characterization, which is dependent on the estimation technique used, is not essential for the development of the proposed technique, and it is outside of the scope of this contribution. The purpose, at this stage, is only to account for the presence of estimation errors in a reasonable manner, noting that the technique can be generalized for any known structure of $\mathbf{w}$.

3. ROBUST WEIGHTED LEAST SQUARES (RWLS) TECHNIQUE

The idea of the proposed approach is to decompose the observation $\mathbf{r}$ in (7) into its known and unknown (random) constituents as follows:

$$\mathbf{r} = b\mathbf{g} \odot \mathbf{e}_\tau + \mathbf{w}'$$

where the new (non-white) noise term $\mathbf{w}'$ includes both the estimation noise and the multipath :

$$\mathbf{w}' = \mathbf{w} + \mathbf{g} \odot \mathbf{e}_\tau \odot \mathbf{u}_{a,D} \tag{8}$$

Then, the WLS joint estimates of τ and b become the minimizers of:

$$F(\tau, b) = \left(\mathbf{r} - b\mathbf{g} \odot \mathbf{e}_\tau\right)^H \mathbf{R}_{\mathbf{w}'}^{-1} \left(\mathbf{r} - b\mathbf{g} \odot \mathbf{e}_\tau\right) \tag{9}$$

where

$$\mathbf{R}_{\mathbf{w}'} = E_{\mathbf{w}} E_{a,D}\left[\mathbf{w}'\mathbf{w}'^H\right] = \sigma_a^2\ \mathbf{g}\mathbf{g}^H \odot \mathbf{Q}_{\sigma_D} \odot \mathbf{e}_\tau \mathbf{e}_\tau^H + \sigma_w^2 \mathbf{I}$$

and the spreading matrix $\mathbf{Q}_{\sigma_D}$ (which has a similar role as the one used in the robust beamforming and detection techniques developed by the authors in [3] and [4], respectively) is defined as follows:

$$\mathbf{Q}_{\sigma_D} = E_D\left[\mathbf{e}_D \mathbf{e}_D^H\right] \tag{10}$$

Note that the covariance matrix $\mathbf{R}_{\mathbf{w}'}$ includes our lack of knowledge about the multipath structure. This constitutes the main idea of the proposed approach. It is not difficult to show that the following property holds:

$$\left(\mathbf{M} \odot \mathbf{e}_\tau \mathbf{e}_\tau^H\right)^{-1} = \mathbf{M}^{-1} \odot \mathbf{e}_\tau \mathbf{e}_\tau^H \qquad (11)$$

Expanding (9), using (11) and (10), the cost function becomes:

$$F(\tau, b) = (\mathbf{r} \odot \mathbf{e}_\tau - b\mathbf{g})^H \mathbf{M}^{-1} (\mathbf{r} \odot \mathbf{e}_\tau - b\mathbf{g}) \qquad (12)$$

where:

$$\mathbf{M} = \mathbf{g}\mathbf{g}^H \odot \mathbf{Q}_{\sigma_D} + \frac{\sigma_w^2}{\sigma_a^2}\mathbf{I} \qquad (13)$$

is now a matrix not dependent on τ. From the definition of $\mathbf{M}$ in (13), we define the Noise-to-Multipath-Ratio as:

$$NMR = \frac{\sigma_w^2}{\sigma_a^2} \qquad (14)$$

In the simulation results of the next section, we will distinguish between NMR (corresponding to the actual scenario) and NMR_{as} corresponding to the assumed value. NMR_{as} is an important design parameter of the technique, and it will have a major effect on the trade-off between bias and variance amplification. Note that NMR_{as} determines the relative importance assigned to the estimation noise $\mathbf{w}$ with respect to the amount of multipath interference. Similarly, we will distinguish between σ_D and $\sigma_{D_{as}}$.

As function $F(\tau, b)$ in (12) is quadratic in b, we can solve for the optimal b given τ, which is:

$$\widehat{b}_\tau = (\mathbf{r} \odot \mathbf{e}_\tau)^H \mathbf{M}^{-1}\mathbf{g} / \left(\mathbf{g}^H \mathbf{M}^{-1} \mathbf{g}\right) \qquad (15)$$

Then, replacing it back into (12), leads to following compressed cost function consisting of two terms:

$$F_c(\tau) = F_1(\tau) + F_2(\tau) \qquad (16)$$

where:

$$F_1(\tau) = (\mathbf{r} \odot \mathbf{e}_\tau)^H \mathbf{M}^{-1} (\mathbf{r} \odot \mathbf{e}_\tau) \qquad (17)$$

$$F_2(\tau) = -\frac{\left|(\mathbf{r} \odot \mathbf{e}_\tau)^H \mathbf{M}^{-1}\mathbf{g}\right|^2}{\mathbf{g}^H \mathbf{M}^{-1} \mathbf{g}} \qquad (18)$$

Assuming that D is a parameter with exponential prior,

$$p(D) = \frac{1}{\sigma_D} e^{-D/\sigma_D} \qquad (19)$$

the elements of the spreading matrix $\mathbf{Q}_{\sigma_D}$ are given by:

$$[\mathbf{Q}_{\sigma_D}]_{p,q} = \int_0^\infty p(D) e^{-(\omega_p - \omega_q)D} = \frac{1}{1 + j(\omega_p - \omega_q)_{\sigma_D}} \qquad (20)$$

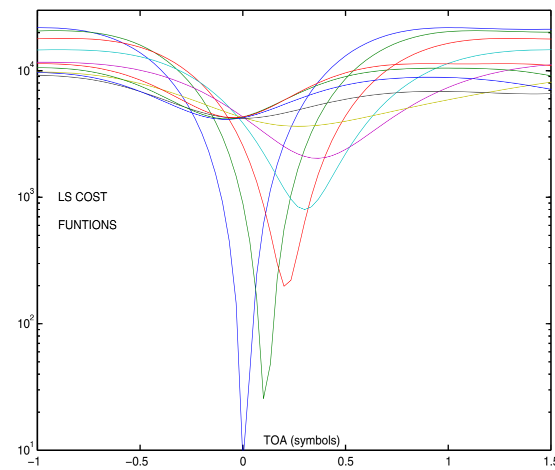

Figure 1: Noiseless LS cost functions.

Finally, note that no assumptions about the prior of a have been required. Only its second order statistics (σ_a^2) is used in our formulation.

4. RESULTS

In this section we try to highlight the performance of the proposed technique in terms of the trade-off between bias reduction and variance amplification in the resulting LOS-TOA estimate (normalized to the symbol period). It is important to emphasize that the amplification of the variance can always be compensated for by averaging several independent TOA estimates (at the price of taking more time for positioning), while the bias does not decrease with averaging. This is why reducing bias at the expense of amplifying variance is of interest. A square-root Nyquist pulse with roll-off 0.35 is considered along with its corresponding matched filter at the receiver. Finally, it is noted that all the TOA values indicated in this section are normalized to the symbol rate.

First, figures 1 and 2 compare the shape of the LS (classical) ($F_c(\tau)$ for $NMR_{as} = \infty$) and WLS ($\sigma_{D_{as}} = 0.5$ and $NMR_{as} = -30dB$) noiseless cost functions in the presence of a single second path of amplitude $a = 0.8$, with increasing delay from $\tau_2 = 0$ up to $\tau_2 = 2$ symbols, and $\tau_1 = 0$. As it can be seen, the minimum of the LS cost function is displaced at right (most of the times) for delays of the second path within the natural time resolution of the pulse shape (related with the inverse signal bandwidth). On the contrary, the WLS function is capable of cancelling significantly the

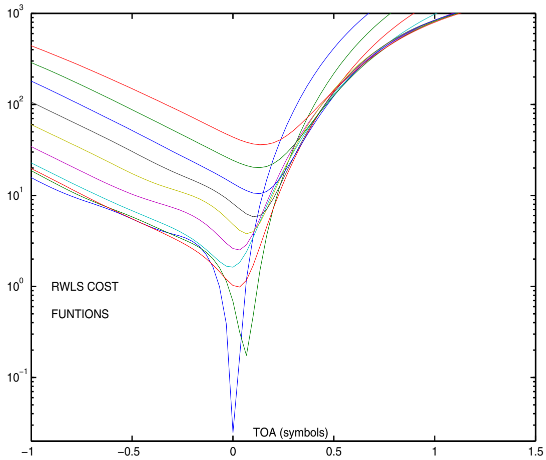

Figure 2: Noiseless WLS cost functions.

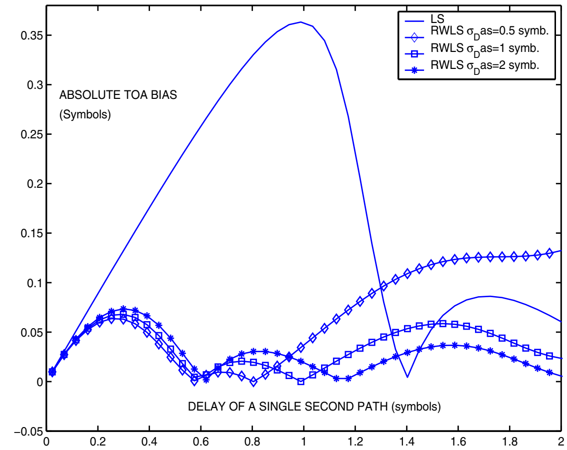

Figure 3: Absolute TOA bias for the single second path secenario.

displacement of its minimum, thus reducing the estimation bias.

Figure 3 shows the evolution of the absolute bias of the LS and WLS cost functions as a function of the delay of a single second path, in the case of $NMR = -35$dB and $NMR_{as} = -30$dB. It can be seen that the effect of increasing $\sigma_{D_{as}}$ is to reduce the bias for a larger range of second delays, at the expense of admitting more bias for small ones. Figure 4 shows the variance associated with the scenarios of figure 3. One can see that the price of the bias reduction is an increased variance with respect to the LS technique.

Finally, figures 5 and 6 show the average bias and average variance amplification. The average is performed over the exponential prior of parameter σ_D for the actual second path delays. These average performance measurements are evaluated as a function of the NMR_{as}, for $\sigma_{Das} = 1$ and different values of σ_D. Both figures include also an scenario with diffuse multipath consisting of ten paths uniformly distributed from $\tau_2 = 0$ to $\tau_2 = 1$ of amplitude $a_m = 0.1$, showing in that case that a similar trade-off between bias and variance takes place. Therefore, it is demonstrated that the use of the proposed approach with a simple multipath structure (only two paths) as a design criterion, also accounts for diffuse multipath in a robust manner.

5. CONCLUSIONS

We have presented a TOA estimation technique which is robust to the presence of unknown diffuse multipath. This property is accomplished without requiring channel estimation, by using a statistical model of the multipath. The reduction of the estimation bias is significant, and it is accomplished at the expense of an increased variance of the resulting TOA estimate. The trade-off between bias reduction and variance amplification can be easily controlled by the hyper-parameters of the model. The increased variance can be easily compensated for by using more data to deliver the final TOA estimate or, in other words, by spending more time to deliver the final mobile position.

Future work will focus on the extension to GMSK signals using the Laurent decomposition [6], following the same ideas as in [7], and to the extension to fast fading channels.

6. REFERENCES

[1] Jürgen Winter and Christian Wengerter. High Resolution Estimation of the Time of Arrival for GSM Location. In *Vehicular Technology Conference Proceedings, VTC-2000*, volume 2, pages 1343–1347, 2000.

[2] Marilynn P. Wylie and Jack Holtzman. The Non-Line of Sight Problem in Mobile Location Estimation. In *5th IEEE International Conference on Universal Personal Communications*, volume 2, pages pp. 827–831, 1996.

[3] Jaume Riba, Jason Goldberg, and Gregori Vázquez. “Robust beamforming for interference rejection in mobile communications”. *IEEE Transactions on*

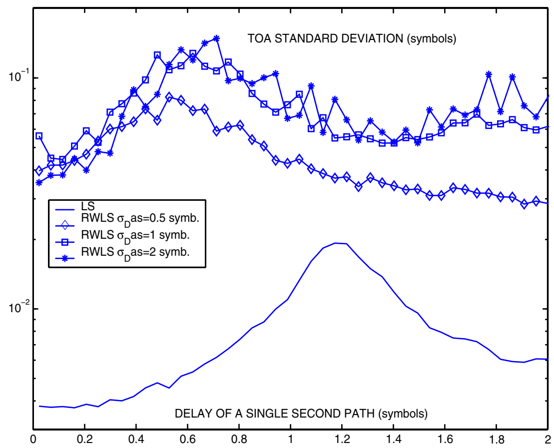

Figure 4: TOA standard deviation for the single second path secenario.

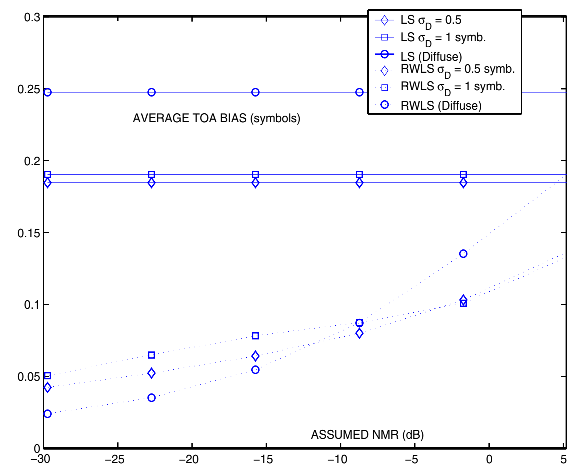

Figure 5: Bias of the WLS technique for different scenarios, as a function of the assumed NMR. LS bias is also ploted for comparison.

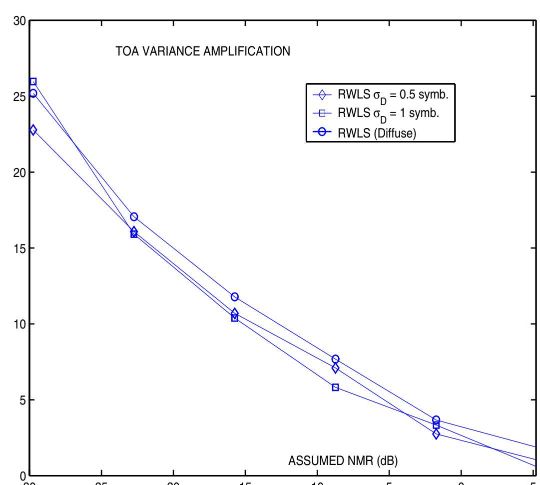

Figure 6: Variance amplification of the WLS technique (with respect to LS) for different scenarios, as a function of the assumed NMR.

Signal Processing, vol. 45(1): pp. 271–275, January 1997. Special Issue on Signal Processing for Advanced Communications.

[4] Jaume Riba, Jason Goldberg, and Gregori Vázquez. “Robust data detection in asynchronous DS-CDMA in the presence of frequency selective fading”. In *Proc.* 8th *IEEE Signal and Array Processing, Corfu (Greece)*, pages pp. 521–524, June 1996.

[5] Klaus Ingemann Pedersen, Preben E. Mogensen, and Bernard H. Fleury. A Stochastic Model of the Temporal and Azimutal Dispersion Seen at the Base Station in Outdoor Propagation Environments. *IEEE Transactions on Vehicular Technology*, 49(2): pp. 437–447, March 2000.

[6] Pierre A. Laurent. “Exact and Approximate Construction of Digital Phase Modulations by Superposition of Amplitude Modulated Pulses”. *IEEE Transactions on Communications*, 34(2): pp. 150–160, February 1986.

[7] Gregori Vazquez and Jaume Riba. *Signal Processing Advances in Wireless and Mobile Communications*, by G.B. Giannakis, Y. Hua, P. Stoica and L. Tong, volume 2 (Trends in Single- and Multi-User Systems). Prentice Hall, July 2000. Chapter 9: Non-Data-Aided Digital Synchronization.