A NOVEL ESTIMATOR AND THEORETICAL LIMITS FOR IN-CAR RADIO-LOCATION

Andreu Urruela and Jaume Riba

Department of Signal Theory and Communications. Universitat Politècnica de Catalunya.
Campus Nord, Ed. D5, Jordi Girona 1 i 3, 08034, Barcelona (Spain).
Email: {andreu,jriba}@gps.tsc.upc.es

I. INTRODUCTION

A problem of growing importance in mobile communication networks is estimating the mobile position since the international commissions announced the minimum requirements in location for the next wireless generations and due to the recent interest for the location-based services. One of the most interesting application in the field of location-based services is the car-routing application where several navigation systems can assist the mobile positioning system with information such inertial system (INS), speed of the mobile or direction of the mobile (compass).

The most well-known method for the position estimation in wireless systems is performed by measuring the time-of-arrival (TOA) or time-difference-of-arrival (TDOA) measurements amongst a set of references (normally BTS's) [1],[2]. The major problem of these kind of methods is the non-linear relationship between measurements and estimation parameters or the difficult problem of finding the intersection of circle or hyperbolas. Several approaches of linearizing the equation system or simplifying the intersection problem have been presented as in [3], where the problem was transformed into a linear intersection equation. The performance of methods based on TOA or TDOA's has been widely studied in recent publications as in [4] where the mobile was always assumed to be placed in a fix position.

This article presents a new method based on TOA measurements that exploits the fact that the mobile is moving with a drift velocity, generalizing the work presented in [5]. The associated lower bound for this new scenario will also be presented in the paper. The numerical simulation shows that the proposed algorithm attains the theoretical limits under certain conditions. The new technique presented opens the door to future low complexity algorithms that exploit a partial or full knowledge of the mobile movement based on the available navigation information (speed, direction, etc).

II. PROBLEM DEFINITION

The problem that we want to solve is finding a simple algorithm to estimate the mobile position when K TOA's measurements are obtained from a set of BTS's and the speed of the mobile and its direction is provided by additional systems. The proposed algorithm solves the non-linear relationship mentioned in the introduction by transforming the observed TOA measurements and obtaining a linear model between the observed transformed measurements and a set of parameters related with the position. After some assumptions about the transformed noise and several geometric interpretation, it is possible to find a simple algorithm that computes the position in a simple way and attaining the theoretical limits. The major advantages of this new algorithm are the low complexity implementation, the lack of initialization (very common in the classic Taylor series based location algorithms) and the possibility of computing the position with a unique BTS (some simple ambiguities have to be solved with additional information in this case)

III. PROPOSED ESTIMATOR

This section presents the main theoretical basis of the proposed estimator based on TOA measurements. For space reasons, several steps in the development will be dropped. First, lets us define TOA measurement taken at the k-th instant of time (t_k) for a certain BTS:

$$t_k = f_k(\mathbf{x}, \mathbf{s}) + w_k = \|\mathbf{x} + k\mathbf{s} - \mathbf{x}_b\| + w_k \qquad (1)$$

where $-\frac{K-1}{2} < k < \frac{K-1}{2}$ is the temporal subindex inside the observation window of length K, $\mathbf{x}$ is the position of the mobile at the middle of the observation window, $\mathbf{x}_b$ is the position of the BTS and $\mathbf{s}$ is the speed constant vector of the mobile (the constant speed assumption is common in in-car applications). Note that since the TOA measurements are obtained with a uniform sampling, the actual speed vector $\mathbf{s}_r$ in m/s is related with the $\mathbf{s}$ vector as $\mathbf{s}_r = \mathbf{s} \cdot r$ where r is the sampling rate.

The proposed estimator is based on the model obtained with the squared TOA measurements. From (1) we have that

$${t_k}^2 = \|\mathbf{x} + k\mathbf{s} - \mathbf{x}_b\|^2 + {w_k}^2 + 2f_k(\mathbf{x}, \mathbf{s})w_k \qquad (2)$$

In fact, ${w_k}^2$ is a random process with mean σ_k^2 so, we can split the two noise terms of (2) into a constant term σ_k^2 and a zero-mean term $w'_k = {w_k}^2 + 2f_k(\mathbf{x}, \mathbf{s})w_k - \sigma_k^2$. Using this notation, and after straightforward operation from (2) we have:

$$t'_k = {t_k}^2 - \sigma_k^2 \approx k^2 \left\|\mathbf{s}\right\|^2 + 2k \left\langle \mathbf{d}, \mathbf{s} \right\rangle + \left\|\mathbf{d}\right\|^2 + w'_k \quad (3)$$

where $\langle \mathbf{v}_1, \mathbf{v}_2 \rangle$ stands for the inner product between two vectors and $\mathbf{d} = \mathbf{x} - \mathbf{x}_b$ is the vector from the BTS to the mobile position in the middle of the observation window. This expression shows us that the new transformed measurements (t'_k) follow a quadratic evolution with the temporal subindex k and are corrupted by zero-mean independent noise terms. This quadratic evolution allows the proposed algorithm to extract explicitly the parameters of the model (related with the position of the mobile) without any kind of initialization or iterative method.

The proposed algorithm is, in fact, based on the application of the well-known weighted Least Square (WLS) method to the transformed measurements (t'_k) in (3) to extract the model parameters and, after several geometrical interpretation of these parameters, computing the position of the mobile in a straightforward manner.

As it is well known, the WLS algorithm is optimum if the linear model is corrupted by Gaussian noise terms. If we assume that the original TOA measurements in (1) are corrupted by Gaussian noise samples (as in [4]) the noise terms of the quadratic model (3) tends to be Gaussian for low values of σ_k^2. In the case of higher values of the noise variance σ_k^2, the WLS can not be considered optimum but practically attains the theoretical limits as numerical simulation show.

The proposed estimator attains the theoretical limits but, at the moment, it has been only presented for the case of one BTS. In the case that multiple BTS's are used, the WLS-based estimator is applied to each BTS to obtain a partial position estimate and the final position estimate ($\widehat{\mathbf{x}}$) is obtained by hybridizing the different partial estimates ($\widehat{\mathbf{x}}_l$ for the l-th BTS) using an estimation of the error covariance matrix ($\widehat{\mathbf{R}_l}$ for the l-th BTS) as follows:

$$\widehat{\mathbf{x}} = \left[\sum_l \widehat{\mathbf{R}_l}^{-1}\right]^{-1} \sum_l \widehat{\mathbf{R}_l}^{-1} \widehat{\mathbf{x}}_l \quad (4)$$

Note that each $\mathbf{R}_l = E\left[\widehat{\mathbf{x}}_l - \mathbf{x}\right]\left[\widehat{\mathbf{x}}_l - \mathbf{x}\right]^T$ has to be estimated directly from the TOA measurements in a real application. Numerical simulations will show that in this case the algorithm attains the theoretical limit asymptotically for large enough observation windows.

IV. SIMULATIONS

The numerical simulation presented here, compare the performances of the proposed algorithm with the theoretical limits as a function of the time observation window (for space reasons, this theoretical limit has not been shown in this extended abstract). The simulation is performed using four BTS's (BTS's positions are [500,0] , [0,500],[-500,0] and [0,-500] in meters and the mobile is placed in the point [0,0]) and with different noise variances (all TOA noise measurements of all BTS's have the same variance and are taken at a rate of 1 sample/second). It can be shown in the figure that with a low noise variance, the algorithm attains the theoretical limit. If we increase the variance of the TOA noise, it can be observed the degradation of the algorithm due to a deficient estimation of the individual error covariances matrices $\widehat{\mathbf{R}_l}$ in the hybridization process shown in (4). On the other hand, it can be seen too, that the algorithm attains asymptotically the theoretical limits for large enough observation windows. In the full paper, it will be also included the analysis when only the modulus of the speed is known.

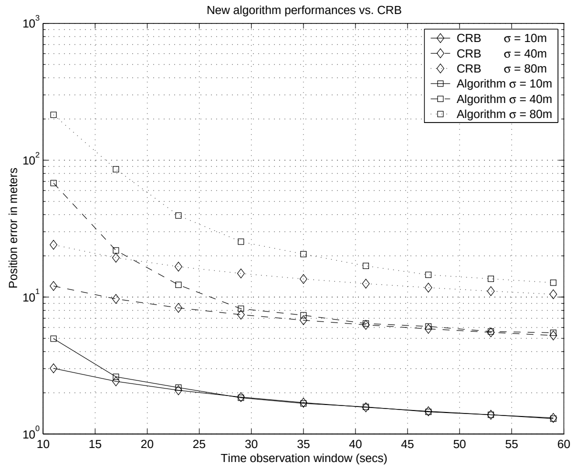

Fig. 1. Comparison between the theoretical limits (CRB) and the algorithm performances

V. REFERENCES

[1] Y.T. Chain and K. Ho. A simple and efficient estimator for hyperbolic location. In *IEEE transactions on signal Processing*, volume 42, Agust 1994.

[2] D.J Torrieri. Statistical theory of passive location sytems. In *IEEE transactions on Aerospace and Electronic Systems*, volume AES-20, March 1984.

[3] J. Jr. Caffery. A new approach to the geometry of TOA location. In *Vehicular Technology Conference, 2000. IEEE VTS-Fall VTC 2000. 52nd , Vol. 4*.

[4] Maurizio A. Spirito. On the accuracy of cellular mobile station location. *IEEE Transactions on Vehicular Technology*, 50(3): pp. 674–685, May 2001.

[5] A. Urruela and J.Riba. A novel estimator and performance bound for time propagation delay and doppler based radio-location. In *International Conference on Acoustics, Speech, and Signal Processing*, April 2003.