

DATA COMPRESSION ALGORITHM IN LOCALIZATION PROBLEMS

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ABSTRACT

This paper presents a flexible scheme for implementing a location algorithm based on TDOA measurements. Concretely, the scheme presented exploits the fact that these measurements are not necessary continuously available, performing a flexible implementation. A general loose coupling approach with other additional sensors and/or position references is here presented.

1. INTRODUCTION

A problem of growing importance in mobile communication networks is estimating the position of mobile terminals. One of the most common approach is getting a position estimation by measuring several time differences of arrival (TDOA) measurements amongst a set of references which are normally base stations and/or satellites. The major problem in this approach is the non-linear relationship between measurements (TDOA) and position parameters (Cartesian coordinates).

In the recent literature, there have been presented several approaches to find a close-form solution as in [3] where the number of measurements equals to the number of unknowns. Several transformations were proposed in [2] and [4] where the non-linear equation system was linearized. The most common approach in this kind of applications is the iterative method [6] where position estimate is improved at each step by determining the local linear Least Square (LS) solution. The major problem of this approach is the high computational cost of the LS procedure due to the fact that a big number of references can be available at each iteration. Moreover, these references can appear and disappear from one iteration to another due to multipath, fading and non line of sight (NLOS) effects. Normally this is not taken into account in the classical approaches.

This work is partially supported by the European Commission under IST Project EMILY¹ focused on the coupling of terrestrial positioning named E-OTD (GSM network) with satellite-based positioning named GNSS in the context of handset solution. In this application, every TDOA (between satellites, between base stations and between the

serving base station and any satellite) is a reference used to improve accuracy of position estimation as well as availability of location-based services. In this context, the number of references presented at the input of an iterative algorithm can be huge, so computational cost of a generic implementation may be therefore very high.

This paper presents a simple and flexible scheme to hybridize all available references presented at the input of each iteration, exploiting the fact that not all references (TDOA) are necessarily available at each iteration. The second objective of this paper is presenting a generic *loose* coupling of independent references in order to simplify the addition of external sensors that can improve the position accuracy as well as improve the stability of the algorithm.

2. CLASSIC IMPLEMENTATION (EKF)

Figure (1) shows the classical implementation of the location algorithm where several groups of independent data are available at the input. These groups normally consist on noise-correlated measurements of TDOAs. Without any lack of generality we will assume that groups of measurements present a null correlation between them. In the case that no-null cross-correlation exists, a unique group will be considered in this model. As it has been mentioned in the introduction, these references may not be available at certain iterations and, moreover, these references can be delivered to the hybridization algorithm (Extended Kalman Filter, EKF) at different rates. These important remarks are not exploited with this classical implementation. However it will be later shown how they can be taken into account with the purposes presented before.

Logically, these measurements groups have to be provided with a quality report to the hybridization block (EKF) that consists, in most cases, on the covariance measurement matrix. If one of these groups is not available at one iteration, the EKF algorithm can iterate without it, taking as a covariance matrix of that n-th group $R_n = \sigma I$ with a big value for variance σ . This produces numerical instability and a big number of useless operations that the classical EKF implementation can not avoid.

In order to understand the mathematical development of the simplified scheme, now we summarize the equations of the EKF implementation. All available data at the input is divided into k groups of independent measurements $r = [r_1^T, r_2^T, \dots, r_k^T]^T$ (this is the measurement vector in the Kalman terminology). The covariance matrix

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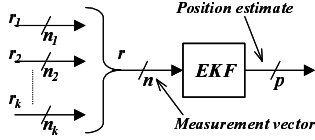


Figure 1: Classical Scheme with EKF

of this global measurement vector is a block-diagonal matrix formed with the individual covariance matrices of each measurement group: $R_k = \text{diag} \left(\begin{bmatrix} R_1 & \cdots & R_k \end{bmatrix} \right)$.

Due to the fact that the relationship between the state vector x (that is the Cartesian coordinates of the estimated position) and the measurement vector r is not linear, it has to be linearized at each iteration with the first time-derivative of this relationship (matrix C in the following expression):

$$r = f(x) + n \approx f(\hat{x}_o) + C\epsilon + n \quad (1)$$

where \hat{x}_o is the previous position estimate and ϵ is the state update vector ($\hat{x} = \hat{x}_o + \epsilon$). If, for instance, our measurement vector is formed with one TDOA reference, this non-linear relationship and its first derivative in time can be expressed as it follows:

$$f(x) = \sqrt{\sum_{i=1}^p (x_i - x_i^n)^2} - \sqrt{\sum_{i=1}^p (x_i - x_i^{n+1})^2} \quad (2)$$

$$C = \begin{bmatrix} \frac{df(x)}{dx_1} & \cdots & \frac{df(x)}{dx_p} \end{bmatrix} \quad (3)$$

where x_i is the i -th Cartesian coordinate of the previous position (\hat{x}_o) estimate, x_i^n is the i -th coordinate of the n -th base station position and p is the dimension of the location problem ($p = 2$ in 2D and $p = 3$ in a 3D location problem). These expressions can be easily generalized if n TDOA measurements are presented at the input. Finally, the variance of the position update vector ϵ of this linearization approach can be written as it follows [5]:

$$R_\epsilon = (C^T R_n^{-1} C)^{-1} \quad (4)$$

The implementation of this approach using the EKF equations can be summarized in the following expression (expression of one iteration).

$$\hat{x} = \hat{x}_o + \epsilon = \hat{x}_o + K(r - f(\hat{x}_o)) \quad (5)$$

where K is the EKF gain and can be computed with the following expression.

$$K = \Sigma (C \Sigma C^T + R^{-1})^{-1} \quad (6)$$

where Σ stands for the covariance state matrix in the KF model. Note that no special emphasis in exploiting the division of the measurement vector is performed in this approach and a low flexibility is offered if one of the input groups is not really available at the current iteration.

3. DATA COMPRESSION AND SUFFICIENT STATISTICS

This section focuses on developing the expressions needed to later present the general scheme of hybridization. For this purpose, let's assume a generic scheme with only one measurements group of size $n > p$, where p is again the state vector dimension. Applying the matrix inversion lemma formulated in [1] to (6), a new expression of the Kalman gain can be found:

$$K = \left[\Sigma (\Sigma + (C^T R^{-1} C)^{-1})^{-1} \right] \left[(C^T R^{-1} C)^{-1} C^T R^{-1} \right] \quad (7)$$

where the right term can be found to be the WLS solution (ϵ_{WLS}) of the linearized equations shown in (1) and the left term is the gain of a linear KF where the transition matrix is I (identity matrix) and the measurement covariance matrix have been transformed into $\tilde{R} = (C^T R^{-1} C)^{-1}$. Note that the WLS term reduces the dimension of the measurement vector to the dimension of the state vector p , so $\epsilon_{WLS} = (C^T R^{-1} C)^{-1} C^T R^{-1} (r - f(x_o))$ turns out to be a sufficient data statistic of measurements. In fact, the left term is a simple KF that implements a loose coupling between the previous position estimate and the new position one with the WLS approach. In this way, the reasoning already made can be easily seen with the following expressions. First of all let's rewrite the EKF iteration equation (5) with these new considerations:

$$\hat{x} = \hat{x}_o + K' \epsilon_{WLS} \quad (8)$$

Now, if we take into account that $\epsilon_{WLS} = \hat{x}_{WLS} - \hat{x}_o$, the previous expression is transformed into a loose coupling between the WLS position estimation and the previous iteration estimate. It can be proved by evaluating the individual coefficients ($I - K'$) and K' in the following expression.

$$\hat{x} = (I - K') \hat{x}_o + K' \hat{x}_{WLS} \quad (9)$$

Recovering the previous result formulated in equation (7), it can be easily generalized for multiple independent data groups, only applying the WLS approach (sufficient data statistic) to every group for compression and then implementing a global loose (as it has been shown for one-group example) with a linear Kalman filter.

The advantages of this approach are mainly two. The first one, a group with a big number of references can be compressed to a state-vector size before feeding the hybridization block, and the second one, if one of these measurements blocks is not really available can be easily removed. The main disadvantage of our presented scheme is the fact that the dimension of the measurement vector of a group has to be at least the dimension of the state vector, because if this condition is not satisfied, the WLS coefficient is not defined (the inverse matrix $(C^T R^{-1} C)^{-1}$ does not exist). It will be latter shown how to solve this problem.

Normally TDOA measurements are independent, so each individual measurement could form an independent group, allowing us to eliminate the individual contribution of each measurement if this is not really available (note that a group formed with only one TDOA measurement never satisfies

the condition presented in the previous paragraph). The objective now is finding, in a flexible manner, a data compression scheme where measurements groups of data will be hybridized without any limitation of the vector dimension. In fact, the final objective will be that any individual independent TDOA measurement (or other kind of measurements) can be eliminated easily from the hybridization algorithm (in the case that this measurement is not available) without any numerical instability and reducing the complexity of the implementation.

As it can be seen in [5], $C^T R^{-1} (r - f(x_0))$ is also a sufficient statistic measurements. In fact, the KF gain (K') of the previous result formulated in (8) can be rewritten as it follows.

$$K' = (\Sigma^{-1} + (C^T R^{-1} C))^{-1} (C^T R^{-1} C) \quad (10)$$

With this new version of the coefficient of the Linear KF, the estimation equation shown in(5) can be rewritten as follow:

$$\hat{x} = x_0 + \epsilon = x_0 + (\Sigma^{-1} + C^T R^{-1} C)^{-1} C^T R^{-1} (r - f(x_0)) \quad (11)$$

Now, the non-defined inversion presented in the previous data compression is transformed into the inverse of the covariance matrix (4) that is perfectly defined. Now, if the measurement vector r is divided into K noise-independent sub-vectors, as we formulated in the previous section, the last expression will be generalized as follow:

$$\hat{x} = \hat{x}_0 + \epsilon = \hat{x}_0 + \left(\Sigma^{-1} + \sum_{k=1}^K \tilde{R}_k^{-1} \right)^{-1} \sum_{k=1}^K C_k^T R_k^{-1} (r_k - f_k(\hat{x}_0)) \quad (12)$$

where $\tilde{R}_k^{-1} = C_k^T R_k^{-1} C_k$ is the inverse of covariance matrix of the individual error position estimation performed with the measurements of group k , that is

$$\hat{x}_k = C_k^T R_k^{-1} (r_k - f_k(x_0)) \quad (13)$$

The last section of this paper will focus on the importance of this inverse covariance matrix used to provide the GDOP report of individual estimates. Finally, it can be developed similarly the expression for the covariance state matrix evolution:

$$\Sigma_{n+1}^{-1} = \Sigma_n^{-1} + \sum_k \tilde{R}_k^{-1} \quad (14)$$

The scheme suggested by this last expression is depicted in figure (2). It can be observed that individual measurement groups are compressed/expanded in order to provide an individual position estimate that is hybridized later in an easy loose manner with the previous global position estimate. If the dimension of the measurement vector is higher than the state vector dimension, the measurement vector is compressed (sufficient statistic), but if the dimension of the measurement vector is smaller than the state vector dimension (for example, an individual independent TDOA), it will be expanded in order to provide an individual incomplete position estimation (see section [5] for a full explanation).

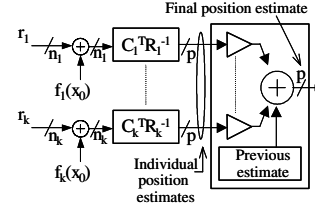


Figure 2: Proposed flexible scheme

4. GENERAL LOOSE COUPLING

In the previous section, the general scheme for hybridization has been introduced. As it can be seen in figure (2), each independent measurement group is compressed/expanded in order to provide a position estimate, and later, a general loose coupling is performed. With this scheme, it is easy to eliminate one measurement block that is not really available at the current iteration. To this end, it is only necessary not to add the contribution of its branch (that is $[C_k^T R_k^{-1} (r_k - f_k(x_0))]$) and not to compute the inverse of its covariance matrix (\tilde{R}_k^{-1}). On the other hand, if measurements groups are delivered at different rates, we can exploit the flexibility of the scheme allowing the use of a certain number of measurement groups (only available groups at that iteration).

Note in the figure, that the individual coefficient of each measurement group transform only the input measurements vector into a position estimate (or error position estimate depending on the final implementation, both options are equivalent). This partial estimate has not to be complete (finite variance in all p dimension), but also can be an incomplete estimate (not full rank covariance inverse matrix: \tilde{R}_k^{-1}). This does not force a minimum number of references to be presented at the input of one measurement group to extract all available information although a loose coupling is later performed. That is, we can use any number of references although a complete position estimation can not be performed with these references.

Finally, additional measurements or other position estimates can be easily added to the scheme. To this end, it is only necessary to provide a partial position estimate and the inverse of the covariance matrix of this estimate (again not necessary full-rank). If additional sensors are available, measurements only have to be transformed (compressed/expanded) into a partial position estimate in the same manner that previous sections have shown. After this transformation, a new branch will be available at the input of the global loose algorithm. If a complete position estimate is available (for example coming from other location techniques), only the inverse of the covariance matrix will have to be computed before being hybridized with the rest of individual estimates. Figure (3) shows the scheme of general loose coupling hybridization with additional sensors.

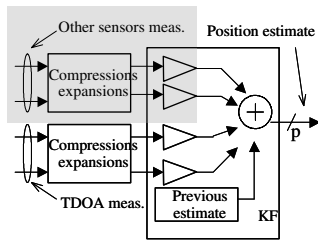


Figure 3: Additional sensors hybridization

5. GDOP CONSIDERATIONS

In the previous section, the general loose hybridization scheme has been shown. In this section, a clear view of the inverse of the covariance matrix will be explained. Each branch in figure (2) provides an error position estimate $C_k^T R_k^{-1} (r_k - f_k(x_0))$, and its quality report: the inverse of the covariance matrix of the estimation ($\tilde{R}_k^{-1} = C_k^T R_k^{-1} C_k$). Note again that this matrix is not full-rank if the dimension of the measurement vector is smaller than the state vector dimension, so the covariance matrix (\tilde{R}_k) is not necessary defined. So, what is the GDOP information presented in the not full-rank inverse matrix used in our scheme?. In a standard location algorithm, the GDOP matrix \tilde{R}_k has to be defined to give a valid position estimate point, but in this case, the individual estimate (each branch of the scheme) has not to be a complete point-defined position estimate.

Let us take a 2D location problem with only one TDOA measurement. In this case, \tilde{R}_k^{-1} is a 2x2 matrix and C_k is a 1x2 matrix. It is easy to verify that \tilde{R}_k^{-1} has rank 1. Then, the covariance matrix is not defined, that is GDOP matrix is infinite. But the dimension of the inverse matrix \tilde{R}_k^{-1} , that is the dimension of measurements vector, is exactly the number of dimensions where the variance of the partial estimation is not infinity. In the previous example, the variance of the individual position estimation $C_k^T R_k^{-1} (r_k - f_k(x_0))$ is infinite in all 2D plane directions except in the perpendicular direction of the hyperbola defined by the TDOA measurement. Figure (4) shows an example with two TDOA measurement that now we comment briefly. The classical concept of GDOP is defined by the following equation.

$$GDOP_k(\alpha) = \begin{bmatrix} \cos \alpha & \sin \alpha \end{bmatrix} \tilde{R}_k \begin{bmatrix} \cos \alpha \\ \sin \alpha \end{bmatrix} \quad (15)$$

Note that this expression is infinite for all values of alpha except the perpendicular direction of the hyperbola. In order to depict GDOP in a graph, the non-defined covariance matrix of the TDOA measurements \tilde{R}_k has been hybridized with a diagonal full-rank matrix ($R_I = kI$) following the loose coupling equation $\tilde{R}_k = (R_I^{-1} + \tilde{R}_k^{-1})^{-1}$, so this new covariance matrix is now full-rank. It can be seen in the graph the maximum reduction of variance in the perpendicular direction of the hyperbola. If now, simulation is repeated (dotted line) with a higher value for k

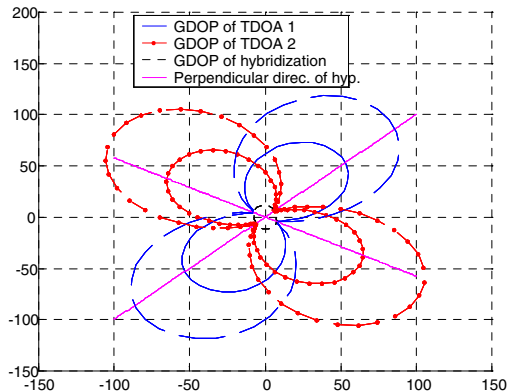


Figure 4: Incomplete GDOP idea and hybridization of GDOPs

(if k is infinite, no previous hybridization approximation is performed), we realize that $GDOP(\alpha)$ tends to infinite in all direction except in the depicted perpendicular direction.

Finally, it is easy to understand that GDOP after the hybridization process will be the superposition of all individual GDOPs (full or not-full rank GDOPs). In the same figure, another rank deficient GDOP has been depicted (corresponding to another hyperbola). If we review equation (14), we will realize that two not-full rank inverse covariance matrices (\tilde{R}_1 and \tilde{R}_2) can perform a complete point-defined estimation ($\tilde{R}^{-1} = \tilde{R}_1^{-1} + \tilde{R}_2^{-1}$ is full-rank).

In this case, there are two incomplete (rank 1) position estimates coming from two TDOA measurements. If one of two TDOA measurements is not available the GDOP information (\tilde{R}_k^{-1}) of the presented TDOA will allow the algorithm continue extracting the available information of the presented measurement at each step.

With this idea in mind, several additional information will be able to be used in the hybridization process. For example, the center of the cell of serving base station can be used as partial estimation with a covariance diagonal matrix proportional of the size of the cell. This will stop an infinite growth of the variance in position estimation in all direction.

6. EMILY ARCHITECTURE

This section presents the current activities of EMILY project, presenting the general scheme of hybridization. In figure (5), a block diagram depicts different parts of the architecture. On the left side of figure, all different kind of available measurements, and on the right, the final position estimate (and speed if it is necessary). The basic Emily architecture is formed with the following measurements: $OTD_{BTS-BTS}$ (TDOA between GSM base stations), Pseudoranges (GPS network) and $OTD_{BTS-SAT}$ (TDOA between serving GSM base station and any satellite under visibility). Other references as O-TDOA (TDOA in UMTS network) and addi-

tional references (inertial sensors, other location techniques, etc..) are also considered in this general scheme.

As it is well known, GSM network is not a synchronous network, so a correction between the different base stations clocks has to be performed. The real time differences between base stations clocks (RTDs in the figure) are reported by the network. These RTD are measured by the network using Location Measurement units (LMU) placed in some GSM cells. In the case of $OTD_{BTS-SAT}$ the meaning of RTD is the time difference between the serving base station clock and general GPS network clock. It is not evident how to measure this time difference due to the different modulation and different rates used. If this measurement can not be reported, the algorithm can perform a complete 2D position with 2 BTSs and 2 SATs, but if $RTD_{BTS-SAT}$ and $OTD_{BTS-SAT}$ can be obtained, full-mode of EMILY architecture can continue performing a complete position with (2 BTSs and 1 SATs) or (1 BTS and 2 SATs). Due to this important benefit, one of the current task in EMILY project is developing an auto-RTD estimate based on a low-variance previous position estimates (based mainly in a good visibility of GPS satellites).

Once all OTD (BTS-BTS and BTS-SAT) have been corrected, a partial position estimate for each one can be achieved with the expressions developed in the previous section (block Error Position estimation in the figure). The rest of measurements (for instance, GPS pseudo-ranges) do not need any previous correction to be projected into a partial position estimate with the correspondant block. The weighted combination hybridizes these different partial position estimates in a loose manner performing a global position estimate. This block is implemented with the decoupled expressions of the KF developed in section (3). The additional sensors and/or other location techniques can be added to this combination only providing a partial position estimation and its quality report (\tilde{R}_k^{-1}). Finally, note that the iterative algorithm have been placed properly (Linear KF) after the combination (not in the combination block) because speed (or higher order model) can be considered. At this final step, speed measurements can be added projecting these measurements to the state vector (Cartesian coordinates of position and speed in this case).

7. CONCLUSIONS

This paper has developed a de-coupled version of the classical extended Kalman filter applied to location problems. Concretely, we have developed the sufficient statistics expressions for compressing or expanding measurements vector (normally TDOA measurements) in order to perform a general loose coupling with the individual position estimates. With this sufficient statistics, we have presented a flexible hybridization scheme that allows us to use an arbitrary number of inputs without any increment of complexity. This scheme is formed by two steps: First, transforming measurement vector in position estimates (or error position estimates) and second, hybridizing these partial position estimates with the previous estimate with a general loose coupling scheme. In this second step, it has been shown the importance of the estimation (or knowledge) of mea-

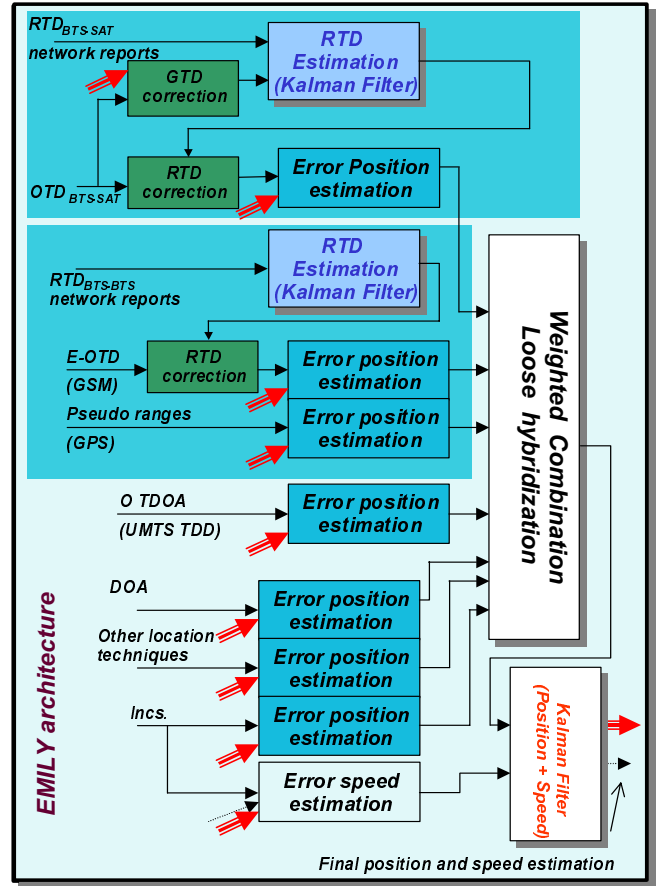


Figure 5: EMILY architecture

surement variance matrix R_k (used in \tilde{R}_k^{-1}). Finally, this simple scheme enables us the addition of external sensors or other possible position references in a flexible manner by adding its contributions in the second step of the scheme.

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