

MINIMUM CONDITIONED VARIANCE (MCV) PROPAGATION DELAY ESTIMATION FOR ASYNCHRONOUS DS-CDMA SYSTEMS

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ABSTRACT

Design accurate estimators which also consider the noise term in low *SNR* scenarios is of paramount importance to achieve optimal solutions. Focusing on asynchronous *DS-CDMA* systems, an accurate estimation of the propagation delays for the different users, is essential to reduce multiple-access interference. The goal of this paper is to propose the *Minimum Conditioned Variance (MCV)* algorithm, resulting an estimator that taking into account the noise impact, improves the *Conditional ML (CML)* solution, and attains the derived *Gaussian Unconditional Cramér-Rao Bound (UCRB)* in the whole *EbNo*. Accordingly, the suggested quadratic estimation technique is shown to be optimal, becoming a great substitute not only to *CML*, but also to *Unconditional ML (UML)* because it achieves similar features in a straightforward way.

Keywords: DS-CDMA; Maximum Likelihood Estimation; Minimum Conditioned Variance (MCV) Estimator; Parameter Estimation; Propagation Delay Estimation; Symbol Detection

1. INTRODUCTION

In a multi-user *DS-CDMA* system, a precise knowledge of the propagation delays is of paramount importance to get a reliable detection of the transmitted symbols at the receiver. Otherwise, as has been widely studied in the literature [1], [2], the performance of the multi-user detector will be rapidly decreased by means of multiple-access interference, and intensely affected by the *near-far* effect.

Maximum Likelihood (*ML*) estimation techniques, have been usually employed to design timing estimators. *Unconditional ML (UML)* and *conditional ML (CML)* algorithms have both been applied in the field

of digital communications. *UML* requires some assumptions on the gaussianity of signal statistics, or assumptions on low *SNR*, to obtain feasible mathematical expressions. While *CML*, applied by Stoica and Nehorai [3] in sensor array processing to perform *DOA* estimation, and more recently also applied to frequency and timing estimation [4]-[7], is not an optimal solution in noisy scenarios with low *SNR*.

Consequently, the restrictions on previous algorithms, motivated the introduction of *Minimum Conditioned Variance (MCV)* method, addressed in this paper. The proposed estimator, solves the *CML* problem at low *SNR* scenarios considering the impact of the noise, and becomes the deterministic solution at high *SNR*. Although the derived *MCV* becomes biased, the bias value can be estimated and next subtracted to obtain an unbiased estimator. The result is an estimator that attains the lower Gaussian *Unconditional Cramér-Rao Bound UCRB* in the whole *EbNo* range, as Gaussian *UML* does. Accordingly, *MCV* becomes an optimal quadratic estimator with no assumptions on the signal statistics. Applications for the proposed *MCV* estimator are further than propagation delay estimation in asynchronous *DS-CDMA* systems, and can also be extended to other estimation problems like frequency synchronization in *OFDM* and *Multi-Carrier* schemes.

This paper is organized as follows. Next section describes a generic discrete-time signal model, which allows to design a *MCV* estimator. Section 3 describes the *CML* formulation and justifies under which conditions the deterministic criterion does not become feasible. Afterwards, section 4 introduces the *Minimum Conditioned Variance* method deriving its gradient expression. Finally, section 5 derives the *UCRB* which is used as a benchmark, at high and low *SNR*'s, and uses this lower bound, to determine the *MCV* performance. Furthermore, simulations establish a comparison between *MCV* and *CML* showing that the proposed estimator outperforms the deterministic one.

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2. DISCRETE-TIME SIGNAL MODEL

The described model considers a K user asynchronous *DS-CDMA* system operating in a multipath environment. The received signal contains the superposition of K active users:

$$r(t) = \sum_{k=1}^K s^k(t - \tau_k) + w(t) \quad (1)$$

where $s^k(t)$ denotes the k -user transmitted baseband signal, τ_k its the propagation delay, and $w(t)$ represents the received AWGN noise term with zero mean and variance σ_w^2 .

For each user the transmitted baseband signal is modeled as:

$$s^k(t) = \sum_{n=-\infty}^{\infty} d_n^k e^{j\theta_k} g^k(t - nT) \quad (2)$$

where $g^k(t)$ represents the k -user transmitted signature, T is the bit duration, d_n^k are the transmitted information bits, and θ_k the received carrier phase. Finally the received signal as a function of the user's signatures is given by:

$$r(t) = \sum_{k=1}^K \sum_{n=-\infty}^{\infty} d_n^k e^{j\theta_k} g^k(t - nT - \tau_k) + w(t) \quad (3)$$

The algorithm is derived in a discrete-time signal model by sampling the received waveform at N_{sc} samples per chip. Choosing the sampling frequency as $f_s = 1/T_s$, where T_s is the sampling period, and collecting $2M + 1$ samples of $r(nT_s)$, the vector \mathbf{r} can be defined as:

$$\mathbf{r} = [r(-MT_s) \quad \dots \quad r(0) \quad \dots \quad r(MT_s)]^T \quad (4)$$

At this point Eq.(3) can be expressed following the matrix signal model:

$$\mathbf{r} = \mathbf{A}(\tau)\mathbf{x} + \mathbf{w} \quad (5)$$

In previous equation, the set of unknown parameters (i.e. the transmitted symbols and phase errors) for k -user define the vector \mathbf{x}^k :

$$\mathbf{x}^k = [d_{-L}^k e^{j\theta_k} \quad \dots \quad d_0^k e^{j\theta_k} \quad \dots \quad d_L^k e^{j\theta_k}]^T \quad (6)$$

where L is related with the number of transmitted symbols $N_s = 2L + 1$. Finally, stacking all users, the nuisance parameter vector \mathbf{x} is defined as follows:

$$\mathbf{x} = [\mathbf{x}^1 T \quad \mathbf{x}^2 T \quad \dots \quad \mathbf{x}^K T]^T \quad (7)$$

On the other hand the model transfer matrix, denoted as $\mathbf{A}(\tau)$ ¹, contains the user signatures, and the parameters to estimate τ_k :

$$\mathbf{A} = \mathbf{A}(\tau) = [\mathbf{A}^1(\tau_1) \quad \mathbf{A}^2(\tau_2) \quad \dots \quad \mathbf{A}^K(\tau_K)] \quad (8)$$

$$\begin{aligned} \mathbf{A}^k(\tau_k) &= [\mathbf{a}_0^k \quad \mathbf{a}_1^k \quad \dots \quad \mathbf{a}_{N_s-1}^k] \\ \mathbf{a}_n^k &= [g^k(-MT_s - nT - \tau_k) \quad \dots \\ &\quad g^k(MT_s - nT - \tau_k)]^T \end{aligned}$$

where the columns of $\mathbf{A}^k(\tau_k)$ are scrolled versions of the k -user signature delayed τ_k .

Notice that the proposed model can consider the impact of a slow time-varying frequency flat fading channel (introducing its influence into vector \mathbf{x}). The only condition is that the channel coefficients remain constant along a symbol period. That is, when the coherence time is higher than the symbol period:

$$\Delta t_c \approx 1/f_d \gg T \quad (9)$$

Finally, a modified discrete-time signal model, including the presence of a propagation multipath channel, can be found in [8].

3. THE CML FORMULATION

The cost function in *CML* estimation for the signal model in Eq.(5) is derived from the joint *ML* cost function that is formulated as:

$$\Lambda(\mathbf{r}/\tau, \mathbf{x}) = \frac{1}{(\pi\sigma_w^2)^M} e^{-\frac{1}{\sigma_w^2} \|\mathbf{r} - \mathbf{A}\mathbf{x}\|^2} \quad (10)$$

The *ML* function depends on the parameter estimation vector τ and also on the vector \mathbf{x} . Notice that vector \mathbf{x} contains the set of unknown parameters and thus it is necessary to take some considerations on this vector. The joint τ, \mathbf{x} estimation could be the solution, but it is discarded because it is computationally complex, and alternative algorithms only focusing on the τ vector estimation are proposed. Classically, *UML* solution computes the expectation of the joint *ML* function with respect to the nuisance parameters:

$$\Lambda_{UML}(\mathbf{r}/\tau) = E_{\mathbf{x}} \{ \Lambda(\mathbf{r}/\tau, \mathbf{x}) \} \quad (11)$$

In general the expectation $E_{\mathbf{x}}$ in Eq.(11) is quite difficult to obtain, and in practice only an approximation of the likelihood function in low *SNR* scenarios is approached.

¹Hereafter the dependence on vector τ will be suppressed for simplicity

Previous limitations motivate the use of the *CML* solution. This method considers the nuisance parameters as deterministic, and thus they can be substituted by its estimation keeping fixed τ vector. The *ML* estimation of \mathbf{x} , when no restrictions are imposed on it, can be obtained as:

$$\hat{\mathbf{x}}_{ML} = \mathbf{A}^\# \mathbf{r} \quad (12)$$

where $\mathbf{A}^\#$ is the Moore-Penrose pseudo-inverse. Once the nuisance vector \mathbf{x} is estimated, the compressed *ML* function to maximize, which only depends on the parameter vector τ , is obtained by replacing Eq.(12) in Eq.(10). And finally the derived log-likelihood function to minimize (omitting irrelevant constants) is given by:

$$\min_{\tau} L_{CML}(\mathbf{r}/\tau) = tr \left\{ \mathbf{P}_A^\perp \hat{\mathbf{R}} \right\} \quad (13)$$

where $\mathbf{P}_A^\perp = \mathbf{I} - \mathbf{A}\mathbf{A}^\#$ is the projection matrix onto the orthogonal subspace defined by \mathbf{A} , and $\hat{\mathbf{R}} = \mathbf{r}\mathbf{r}^H$. To minimize Eq.(13) a gradient algorithm may be used. The gradient in conditional *ML* was derived by Viberg, Ottersten and Kailath [9] in the context of array processing for *DOA* estimation. In our delay estimation problem this gradient can be expressed as:

$$g_{c_i} = -2Re \left\{ (\mathbf{r}^H \mathbf{P}_A^\perp \mathbf{D}_i) (\mathbf{A}^\# \mathbf{r}) \right\} \quad (14)$$

where $\mathbf{D}_i = \frac{\partial}{\partial \tau_i} \mathbf{A}$.

A more accurate study of the gradient expression shows that it is computed by the product of two terms. The first term is $(\mathbf{r}^H \mathbf{P}_A^\perp \mathbf{D}_i)$ and justifies the proposed algorithm to be *self-noise* free. Considering a noiseless environment, and the absence of delay errors, vector \mathbf{r} will be contained in the signal subspace generated by the \mathbf{A} matrix columns. Thus, the projection matrix \mathbf{P}_A^\perp , which does not appear in the classical unconditional approach, acts as a zero-forcer placed at the output of the derivative matched filter \mathbf{D}_i . As a result, the estimator ensures in all cases a *self-noise* free solution: $(\mathbf{r}^H \mathbf{P}_A^\perp \mathbf{D}_i) = 0$.

The second term $(\mathbf{A}^\# \mathbf{r})$ corresponds to the *ML* estimation of the unconstrained vector \mathbf{x} . Notice that this expression is the decorrelating detector solution, so the algorithm not only estimates the propagation delay but also implements this sub-optimum detector. The presence of this term justifies the proposed solution to be a robust *near-far* estimator. Analyzing the signal model Eq.(5) it is observed that the received powers can be introduced in the nuisance parameter vector \mathbf{x} . Hence, following Eq.(12) it is guaranteed that the algorithm will estimate the received power values, justifying the estimator to be insensitive to different power levels. Nevertheless, the decorrelating detector evidences some

difficulties in noisy scenarios. The pseudoinverse, as the ideal zero-forcing solution *ZF* in equalization, does not take into account the noise term. Accordingly, when the transfer matrix \mathbf{A} eigenvalue spreading, defined as:

$$\chi = \frac{\lambda_{A_{max}}}{\lambda_{A_{min}}} \quad (15)$$

is large enough, the noise term will be extremely increased, becoming the *CML* method an unacceptable solution in low *SNR* scenarios, which are common in wideband *DS-CDMA* systems.

4. MINIMUM CONDITIONED VARIANCE APPROACH

A novel approach is proposed in this paper considering the impact of the noise in the likelihood function, achieving in consequence a more robust estimator in low *SNR* scenarios. The *Minimum Conditioned Variance* approach (*MCV*) makes the nuisance parameter estimation as the best linear estimation under a minimum variance criterion given an observation vector \mathbf{r} . This estimation is:

$$\begin{aligned} \hat{\mathbf{x}} &= E[\mathbf{x}/\mathbf{r}] = \mathbf{\Gamma} \mathbf{A}^H (\mathbf{A} \mathbf{\Gamma} \mathbf{A}^H + \sigma_w^2 \mathbf{I})^{-1} \mathbf{r} = \mathbf{C} \mathbf{r} \\ \mathbf{C} &= \mathbf{\Gamma} \mathbf{A}^H (\mathbf{A} \mathbf{\Gamma} \mathbf{A}^H + \sigma_w^2 \mathbf{I})^{-1} \\ \mathbf{\Gamma} &= E\{\mathbf{x}\mathbf{x}^H\} \end{aligned} \quad (16)$$

Previous expression belongs to the best linear and non-linear estimator under Gaussian conditions, and only the best linear estimator under non-Gaussian conditions. The new cost function is derived by substituting Eq.(16) in Eq.(10) and it is given by:

$$\min_{\tau} L_{MCV}(\mathbf{r}/\tau) = \|\mathbf{r} - \mathbf{A}\mathbf{C}\mathbf{r}\|^2 \quad (17)$$

At high *SNR* scenarios $\mathbf{C}(\sigma_w^2 \rightarrow 0) = \mathbf{A}^\#$ is the pseudo-inverse of \mathbf{A} , becoming the *CML* solution established in previous section. On the other hand, when the contribution of $\mathbf{A} \mathbf{\Gamma} \mathbf{A}^H$ is negligible in front of $\sigma_w^2 \mathbf{I}$, \mathbf{C} approaches a bank of matched filters containing all the user signatures: $\mathbf{C}(\sigma_w^2 \rightarrow \infty) = \sigma_w^{-2} \mathbf{\Gamma} \mathbf{A}^H$. This second limit is achieved at low *SNR* when the noise power is much greater than the received signal power for all users. Notice however that, in high *near-far* scenarios, the elements in $\mathbf{\Gamma}$ associated to the most powerful users will be higher than the noise term. Consequently, in scenarios with low *SNR* and small *near-far*, the *MCV* will improve the classical *CML* solution, whereas in high *near-far* scenarios, *MCV* will remain close to *CML*.

To minimize Eq.(17) we will follow once again a gradient scheme. The gradient expression in *MCV* is given by:

$$g_{mcv_i} = -2Re \left\{ \mathbf{r}^H (\mathbf{I} - \mathbf{A}\mathbf{C})^H \left(\mathbf{D}_i \mathbf{C} + \mathbf{A} \frac{\partial}{\partial \tau_i} \mathbf{C} \right) \mathbf{r} \right\} \quad (18)$$

It results interesting to analyze the previous gradient behaviour at high and low *SNR* scenarios. At high *SNR* $\mathbf{C} \rightarrow \mathbf{A}^\#$, and making use of $\mathbf{P}_A^\perp \mathbf{A} = 0$, the second term in the previous gradient is asymptotically equal to zero: $\mathbf{r}^H (\mathbf{I} - \mathbf{A}\mathbf{C})^H \mathbf{A} \frac{\partial}{\partial \tau_i} \mathbf{C} \mathbf{r} = 0$. Thus the gradient becomes:

$$g_{mcv_i} (\sigma_w^2 \rightarrow 0) \simeq -2Re \left\{ \mathbf{r}^H (\mathbf{I} - \mathbf{A}\mathbf{C})^H (\mathbf{D}_i \mathbf{C}) \mathbf{r} \right\} \quad (19)$$

Likewise, at low *SNR*'s $\mathbf{C} \rightarrow \sigma^{-2} \mathbf{\Gamma} \mathbf{A}^H$, and the two components in the gradient Eq.(18) supply the same value. Hence, the asymptotic gradient derived in noisy environments corresponds with:

$$g_{mcv_i} (\sigma_w^2 \rightarrow \infty) \simeq -\frac{4}{\sigma_w^2} Re \left\{ \mathbf{r}^H \mathbf{D}_i \mathbf{\Gamma} \mathbf{A}^H \mathbf{r} \right\} \quad (20)$$

After the previous analysis, that shows the second term can be dropped without losing information by the gradient, the *MCV* gradient can be asymptotically rewritten as:

$$g_{mcv_i} \approx -2Re \left\{ \mathbf{r}^H (\mathbf{I} - \mathbf{A}\mathbf{C})^H \mathbf{D}_i \mathbf{C} \mathbf{r} \right\} \quad (21)$$

Once the gradient has been derived, an important difference between uni-parametric and multi-parametric estimators making use of *MCV* is next introduced. When there is only one parameter to estimate, e.g. timing or frequency estimation in linear and non-linear modulations [7], and considering that vector \mathbf{r} follows a Gaussian distribution, which is known to be a non-realistic assumption in digital communications, the Gaussian *UML* cost function becomes ²:

$$L_{UML^G}(\mathbf{r}/\tau) = \mathbf{r}^H \mathbf{R}^{-1} \mathbf{r} \quad (22)$$

$$\mathbf{R} = \left(\mathbf{A} \mathbf{\Gamma} \mathbf{A}^H + \sigma_w^2 \mathbf{I} \right)$$

and the Gaussian *UML* gradient in previous equation is given by:

$$g_{uml_i} = -2Re \left\{ \mathbf{r}^H (\mathbf{I} - \mathbf{A}\mathbf{C})^H \mathbf{D}_i \mathbf{C} \mathbf{r} \right\} \quad (23)$$

Comparing last equation with Eq.(21), it is shown that assuming a Gaussian distribution for the transmitted

²Only applicable if $\mathbf{A}^H \mathbf{A}$ does not depend on the parameter to estimate

symbols, the *MCV* gradient becomes the Gaussian *UML* gradient. Nevertheless, in multi-parametric estimators, the Gaussian *UML* cost function becomes more complex:

$$L_{UML^G}(\mathbf{r}/\tau) = \ln |\mathbf{R}| + \mathbf{r}^H \mathbf{R}^{-1} \mathbf{r} \quad (24)$$

Notice a new term $\ln |\mathbf{R}|$, which becomes constant in the uniparametric estimators when $\mathbf{A}^H \mathbf{A}$ does not depend on the parameter to estimate, is introduced. The gradient expression, derived in [3] cannot be identified with Eq.(21) anymore. Consequently, multi-parametric *MCV* estimators does not become Gaussian *UML*. However, the proposed *MCV* estimator achieves similar performance than Gaussian *UML* making use of a simple quadratic estimator, as simulations will show.

A more accurate analysis of the previous gradient shows it is biased. It can be seen that in the absence of timing errors the gradient does not become the null vector. Therefore, the bias expression can be obtained computing the gradient expected value when the estimated timing vector equals the real timing vector:

$$\begin{aligned} Bias_i &= E \{ g_{mcv_i} \} |_{\hat{\tau}=\tau} \\ &= -2Re \left\{ Tr \left\{ \mathbf{\Gamma} \mathbf{A}_\tau^H (\mathbf{I} - \mathbf{A}_\tau \mathbf{C}_\tau)^H \mathbf{D}_{i_\tau} \right\} \right\} \end{aligned} \quad (25)$$

denoting with \mathbf{A}_τ , \mathbf{C}_τ , \mathbf{D}_{i_τ} , the dependence of matrices on vector τ .

Unfortunately previous expression cannot be computed by the estimator because the real timing vector τ is not a priori known. Nevertheless, the gradient expected value close to the real timing vector does not depend on the absolute timing error $\tau - \hat{\tau}$. Hence, an accurate bias estimation can be obtained if the estimated timing vector is used to compute Eq.(25):

$$\widehat{Bias}_i = -2Re \left\{ Tr \left\{ \mathbf{\Gamma} \mathbf{A}_{\hat{\tau}}^H (\mathbf{I} - \mathbf{A}_{\hat{\tau}} \mathbf{C}_{\hat{\tau}})^H \mathbf{D}_{i_{\hat{\tau}}} \right\} \right\} \quad (26)$$

As a result, an unbiased estimation of τ vector can be obtained according to a modified gradient where the bias is subtracted:

$$g_{mcv_i}^{unbiased} = -2Re \left\{ \mathbf{r}^H (\mathbf{I} - \mathbf{A}\mathbf{C})^H \mathbf{D}_i \mathbf{C} \mathbf{r} \right\} - \widehat{Bias}_i \quad (27)$$

5. PERFORMANCE ANALYSIS AND SIMULATIONS

This section derives the Gaussian Unconditional Cramér-Rao Bound (*UCRB*) to compare it with the proposed *CML* and *MCV* multi-user delay estimators, analyzing its performance.

Unconditional CRB

As it is shown in [3] the *UCRB* is a valid lower bound for the variance of any consistent estimator based on the data sample covariance matrix.

As derived in [10] the ij th Fisher Information Matrix (*FIM*) element can be obtained as:

$$\{FIM_u\}_{ij} = Tr \{ \mathbf{R}^{-1} \mathbf{R}_i \mathbf{R}^{-1} \mathbf{R}_j \} \quad (28)$$

where:

$$\begin{aligned} \mathbf{R} &= E \{ \mathbf{r} \mathbf{r}^H \} = \mathbf{A} \mathbf{\Gamma} \mathbf{A}^H + \sigma_w^2 \mathbf{I} \\ \mathbf{R}_i &= \frac{\partial}{\partial \tau_i} \mathbf{R} \end{aligned} \quad (29)$$

Focusing on our estimation problem, assuming that the noise power is *a priori* known (which is considered in the *MCV* case), and modeling the transmitted symbols to be zero mean independent random variables (i.e. $\mathbf{\Gamma}$ is a diagonal matrix) \mathbf{R}_i results:

$$\begin{aligned} \mathbf{R}_i &= \sigma_{x_i}^2 \left(\mathbf{D}_i \mathbf{A}^H + \mathbf{A} \mathbf{D}_i^H \right) \quad i = 1 \dots N_s \\ \mathbf{D}_i &= \frac{\partial}{\partial \tau_i} \mathbf{A} \end{aligned} \quad (30)$$

which can be substituted into Eq.(28) to obtain the *UCRB*. If the noise power is not *a priori* known the *UCRB* is derived in [10]:

$$\left\{ FIM'_u \right\}_{ij} = \frac{2\sigma_{x_i}^2 \sigma_{x_j}^2}{\sigma_w^2} Re \left\{ Tr \left\{ \mathbf{D}_i^H \mathbf{P}_A^\perp \mathbf{D}_j \mathbf{A}^H \mathbf{R}^{-1} \mathbf{A} \right\} \right\} \quad (31)$$

It is interesting to study the performance of previous *UCRB* for high and low *SNR*'s.

At high *SNR*'s:

$$R_{\sigma_w^2 \rightarrow 0} \rightarrow \left(\mathbf{A} \mathbf{\Gamma} \mathbf{A}^H \right)^{-1} \quad (32)$$

and thus:

$$\left\{ FIM'_u \right\}_{ij} \rightarrow \begin{cases} \frac{2\sigma_{x_i}^2}{\sigma_w^2} Re \left\{ Tr \left\{ \mathbf{D}_i^H \mathbf{P}_A^\perp \mathbf{D}_j \right\} \right\} & i = j \\ 0 & i \neq j \end{cases} \quad (33)$$

At low *SNR*'s:

$$R_{\sigma_w^2 \rightarrow \infty} \rightarrow \frac{1}{\sigma_w^2} \mathbf{I} \quad (34)$$

and the *FIM* results:

$$\left\{ FIM'_u \right\}_{ij} \rightarrow \frac{2\sigma_{x_i}^2 \sigma_{x_j}^2 Re \left\{ Tr \left\{ \mathbf{D}_i^H \mathbf{P}_A^\perp \mathbf{D}_j \mathbf{A}^H \mathbf{R}^{-1} \mathbf{A} \right\} \right\}}{\sigma_w^4} \quad (35)$$

which implies a performance degradation at low *SNR*'s due to the noise power square term in the denominator as it will be perceived in the simulations.

Performance Comparisons

To evaluate the *CML* Eq.(14) and *MCV* Eq.(27) estimators the Root-Mean Square Error (*RMSE*) in the timing delay estimation was computed. Simulations were done considering 5 users, the spreading codes were Gold sequences with 7 chips per bit, the pulse shaping was a square-root raised cosine pulse with roll-off factor equal to 0.5, the considered modulation was BPSK, and the oversampling factor was $N_{sc} = 2$. The number of transmitted symbols was $N_s = 64$ ³, and two simulations were done for the \mathbf{A} matrix eigenvalue spreading Eq.(15) $\chi = 35$ and $\chi = 65.5$. *Near-far* environments were considered defining the *near-far* ratio as $\max(P_i/P_1) \quad i \neq 1$, and computing the *EbNo* for the the lowest received power user (i.e. user 1).

Figure 1 compares the proposed *MCV* versus the classical *CML* algorithm, and compares the *RMSE* with the derived *UCRB* lower bound assuming that the noise power σ_w^2 is a priori known Eq.(28) - Eq.(30). A low *SNR* scenario with *near-far NF*=0dB was simulated. As it can be seen in figure 1, due to the high eigenvalue spread, at low *SNR* the *CML* is not an optimal solution and does not achieve the derived *UCRB*. Furthermore, the higher the eigenvalue spreading is, the worse the *CML* solution performs. Which is more interesting to analyze is that under the same conditions, the proposed *MCV* outperforms the *CML* algorithm and attains the *UCRB*, becoming a quadratic optimal solution. Figure 1 also illustrates how at high *SNR* the *MCV* becomes the *CML* solution, and asymptotically both attain the Cramér-Rao Bound. Similar results are obtained in figure 2, where a new scenario with *near-far NF*=10dB was simulated. As it can be seen, *MCV* outperforms *CML* once more, and attains again the *UCRB*.

6. CONCLUSIONS

In this paper the *MCV* algorithm has been introduced in the multiuser propagation delay estimation context. This novel method modifies the classical *CML* solution considering the impact of the noise in the Likelihood function compression. Hence, a more robust algorithm in noisy environments when the transference matrix eigenvalue dispersion is large, can be derived.

Simulations have shown *MCV* outperforms the classical deterministic algorithm in noisy conditions, and it corresponds asymptotically with the *CML* at high *SNR*'s. The mean squared timing error and the bit-error rate at the symbol detection have been used to

³This parameter is related with the equivalent noise loop bandwidth B_L as $B_L T = \frac{1}{2N_s}$ [12]

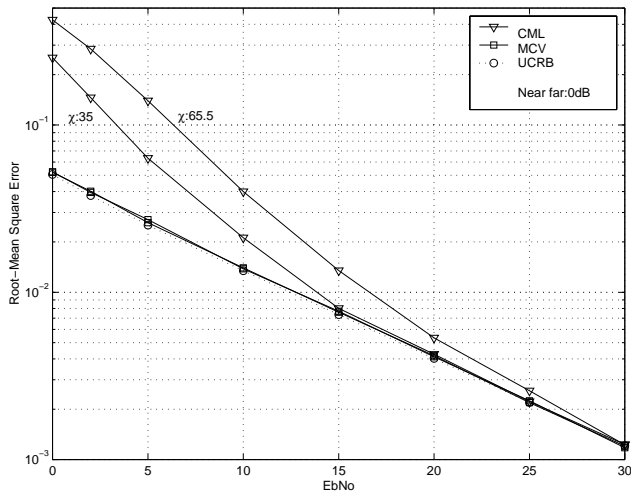


Figure 1: Root-Mean Square Timing Error. $NF:0dB$

evaluate this performance. Accordingly, the suggested quadratic estimation technique is shown to be optimal since it attains the *UCRB* lower bound in the whole *EbNo* range, becoming a great substitute not only to *CML*, but also to *UML* because it achieves similar features in a straightforward way.

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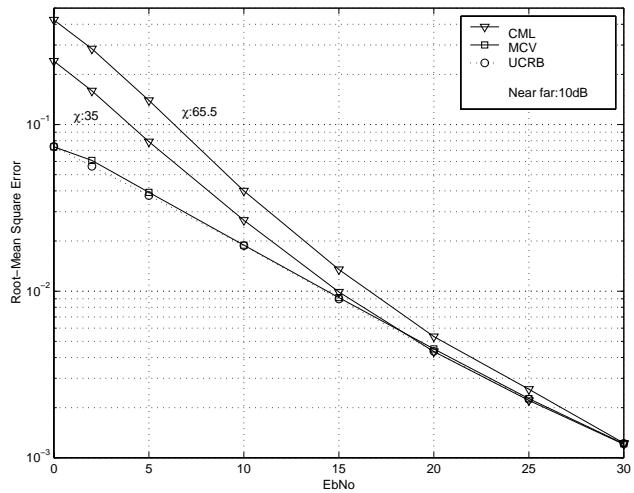


Figure 2: Root-Mean Square Timing Error. $NF:10dB$

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