A Unified Framework for Communications through MIMO Channels

Ph.D. Dissertation

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Outline of Presentation

• Motivation
• MIMO Channels: An Overview
• Capacity of MIMO Channels
• Joint Tx-Rx Design for MIMO Channels
  – Power-Constrained Systems: A Unified Framework
  – QoS-Constrained Systems
• Robust Design against Channel Estimation Errors
• Conclusions
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Motivation

- **Wireless multi-antenna communication systems**
  - Scarce radio spectrum
  - Increasing demand: more users and higher quality
  - Increase spectral efficiency
  - Rapid advance in technology: multiple antennas

- **DSL (wireline) communication systems**
  - Broadband access technology: high data rates over telephone lines: last-mile access.
  - Originally built for telephone service.
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MIMO Channels

- MIMO channels are an abstract way of representing many different types of communication channels:
  - Wireless multi-antenna systems
  - Wireline (DSL) systems

- MIMO systems can be modeled in a compact and elegant way using a vector-matrix notation.

- A MIMO channel has the capability of multiplexing to create a set of parallel substreams.
Signal Model (I)

- Single MIMO channel

\[ y = Hs + n \]

- Simple but powerful signal model.
Examples (I)

- Convolutional channel

$$s(n) \rightarrow h(n) \rightarrow y(n)$$

- Signal model in time domain

$$y(n) = \sum_{k=0}^{L} h(k) s(n-k)$$

- Signal model in frequency domain (multicarrier)

$$y(k) = H\left(\frac{2\pi k}{N}\right) s(k) + n(k) \quad 0 \leq k \leq N - 1$$
Examples (II)

- Representation as a MIMO channel (time domain)

\[ y(n) = \sum_{k=0}^{L} h(k) s(n - k) \]

- Representation as a MIMO channel (freq. domain)

\[ y(k) = H \left( \frac{2\pi k}{N} \right) s(k) + n(k) \quad 0 \leq k \leq N - 1 \]

\[ \Rightarrow \quad y = D_H s + n \]
Examples (III)

- Multi-antenna wireless channel

\[ y(n) = \sum_{k=0}^{L} H(k) s(n - k) + n(n) \]
Examples (V)

- DSL channel (bundle of twisted-pair copper wires)

\[ y(n) = \sum_{k=0}^{L} H(k) s(n - k) + n(n) \]

- Matrix convolution signal model
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Capacity of MIMO Channels

• Fundamental limit of a channel: capacity.

• Capacity depends on many factors: CSI, power limitations, bandwidth limitations, delay constraints.

• Capacity for different degrees of CSIT:
  – Capacity with instantaneous CSIT
  – Capacity with statistical CSIT
  – Capacity with no CSIT: complete uncertainty

  ↓

  Robust Approach
Capacity with Instantaneous CSIT

- The transmitter can adapt to each channel realization

\[ C(\mathbf{H}) = \max_{\mathbf{Q}: \text{Tr}(\mathbf{Q}) \leq P_T} \log \det \left( \mathbf{I}_{n_R} + \mathbf{H} \mathbf{Q} \mathbf{H}^H \right) \quad \mathbf{Q} = \mathbb{E} [\mathbf{x} \mathbf{x}^H] \]

- Optimal \( \mathbf{Q} \) obtained from the SVD of the channel matrix

\[
\mathbf{H} = \mathbf{U} \Sigma \mathbf{V}^H \\
\mathbf{Q} = \mathbf{V} \mathbf{D}_Q \mathbf{V}^H
\]

where \( \mathbf{D}_Q \) follows a water-filling distribution:

\[
\lambda_{Q,i} = (\mu - \lambda_i^{-1})^+ \quad 1 \leq i \leq n_T
\]
Capacity with Statistical CSIT

- The transmitter cannot adapt to each channel realization: it is fixed.

- The channel is treated as a random variable with a (known) pdf.

- The optimal $Q$ can be pre-computed according to:

  - Ergodic capacity

    $$C_{\text{erg}} = \max_{\text{Tr}(Q) \leq P_T} \mathbb{E}_H \log \det \left( I_n + HQH^H \right)$$

  - Outage capacity
Capacity with no CSIT

- What should the transmitter do when it does not even know the channel pdf??
- Robust approach: be ready for the worst scenario.
- The problem can be formulated as a game between two players: the transmitter and nature

\[
\begin{align*}
\text{Player 1} & \quad \text{"Transmitter"} \\
\text{Player 2} & \quad \text{"Nature"} \\
\end{align*}
\]
Maximin Formulation

- Mathematically, the robust approach is expressed as a maximin problem

\[
\max_{Q \in \mathcal{Q}} \min_{H \in \mathcal{H}} \log \det \left( I_{n_R} + HQH^H \right)
\]

- The transmitter is constrained in its average power.
- The channel has to be constrained to avoid the trivial solution, but what constraint should we use??
  - Energy constraint \( \|H\|_F^2 \geq \beta \)
  - Max. singular value constraint \( \sigma_{\max}(H) \geq \beta \)
Uniform Solution: A Robust Solution

- Theorem: The solution to the maximin formulation

\[ Q = \arg \max_{Q \in \mathcal{Q}} \min_{H \in \mathcal{H}} \log \det \left( I_{n_R} + HQH^H \right) \]

is independent of the channel constraint (under a mild condition) and is given by the uniform power allocation

\[ Q = P_T/n_T I_{n_T} \]

- Interpretation: since the channel is unknown, transmit equally and simultaneously in all directions.
Extension to the Multiple-Access Channel

- Game formulation:

  Player 1
  "Transmitter"

  Player 2
  "Nature"

- What is the optimal power allocation for each user in the maximin sense (robust solution) ???
Capacity Region of the MAC

- The rate region for a given set of channels and transmit covariance matrices is

\[
\mathcal{R}(\{Q_k\}, \{H_k\}) = \left\{ (R_1, \ldots, R_K) : \sum_{k \in S} R_k \leq \log \det \left( I_{n_R} + \sum_{k \in S} H_k Q_k H_k^H \right), \forall S \subseteq \{1, \ldots, K\} \right\}
\]

- The worst-case rate region is the set of rates that can always be achieved no matter the channel state

\[
\mathcal{R}(\{Q_k\}, \mathcal{H}) = \bigcap_{\{H_k\} \in \mathcal{H}} \mathcal{R}(\{Q_k\}, \{H_k\})
\]
Uniform Solution: A Robust Solution

- Theorem: The worst-case capacity region is obtained when each user transmits with a uniform power allocation

\[ Q_k = P_k / n_k \mathbf{I}_{n_k} \]

- The worst-case rate region corresponding to the uniform power allocation for each user contains that of any other power allocation strategy:

\[ \mathcal{R} (\{Q_k\}, \mathcal{H}) \subseteq \mathcal{R} (\{P_k / n_k \mathbf{I}_{n_k}\}, \mathcal{H}) \quad \forall Q_k : \text{Tr} (Q_k) \leq P_k \]
MAC: An Example

- Illustration of $\mathcal{R} (\{Q_k\}, \mathcal{H}) \subseteq \mathcal{R} (\{P_k/n_k I_{n_k}\}, \mathcal{H})$
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Signal Model: Single Beamforming (I)

- Scheme of single beamforming (beamvectors) for a multicarrier MIMO channel
Signal Model: Single Beamforming (II)

- Transmitted signal using a beamvector

\[ s_k = b_k x_k \quad 1 \leq k \leq N \]

subject to power constraint

\[ \sum_{k=1}^{N} \|b_k\|^2 \leq P_T \]

- Received signal postprocessed with a beamvector

\[ \hat{x}_k = a_k^H y_k \quad 1 \leq k \leq N \]

- One substream is established per MIMO channel.
Signal Model: Multiple Beamforming (I)

- Scheme of multiple beamforming (beam-matrix) for a single MIMO channel

- Interpretation as multiple beamvectors:
Signal Model: Multiple Beamforming (II)

- Transmitted signal using a beam-matrix

\[ s_k = B_k x_k = \sum_{i=1}^{L_k} b_{k,i} x_{k,i} \quad 1 \leq k \leq N \]

subject to power constraint

\[ \sum_{k=1}^{N} \| B_k \|_F^2 = \sum_{k=1}^{N} \text{Tr} \left( B_k B_k^H \right) \leq P_T \]

- Received signal postprocessed with a beam-matrix

\[ \hat{x}_k = A_k^H y_k \quad 1 \leq k \leq N \]

- Several substreams are established per MIMO channel.
Preliminary: Convex Optimization Theory

- Convex optimization problem

\[
\begin{align*}
\min_{x} & \quad f_0(x) \\
\text{s.t.} & \quad f_i(x) \leq 0 \quad 1 \leq i \leq m
\end{align*}
\]

- Only global solutions (not local).

- Closed-form solutions may be obtained from the KKT optimality conditions.

- If not, the problem can still be solved very efficiently in practice using interior-point methods.
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Traditional Approach (I)

- Choose a particular objective function to measure the performance of the system and then optimize the system accordingly.

- In general, the MSE matrix is used which is defined as

\[
    E_k \triangleq E \left[ (\hat{x}_k - x_k)(\hat{x}_k - x_k)^H \right] \\
    = (A_k^H H_k B_k - I) (B_k^H H_k^H A_k - I) + A_k^H R_{n,k} A_k
\]

where

\[
    \text{MSE}_{k,i} = E \left[ |\hat{x}_{k,i} - x_{k,i}|^2 \right] = [E_k]_{ii}
\]
Traditional Approach (II)

- Minimization of the trace of the MSE matrix (sum of the MSE’s of the substreams):

  $$\min_{A,B} \quad \text{Tr}(E) = \sum_i \text{MSE}_i$$

  s.t. \quad \text{Tr}(B^H B) \leq P_T

  The optimal solution diagonalizes the channel matrix and the MSE matrix.

- Minimization of the trace of the weighted MSE matrix (weighted sum of the MSE’s of the substreams):

  $$\min_{A,B} \quad \text{Tr}(WE) = \sum_i w_i \text{MSE}_i$$

  s.t. \quad \text{Tr}(B^H B) \leq P_T

  The optimal solution diagonalizes the channel matrix and the MSE matrix.
Traditional Approach (III)

- Minimization of the determinant of the MSE matrix:

\[
\min_{\mathbf{A}, \mathbf{B}} |\mathbf{E}|
\]
\[
\text{s.t.} \quad \text{Tr} (\mathbf{B}^H \mathbf{B}) \leq P_T
\]

The optimal solution diagonalizes the channel matrix and the MSE matrix.

- Maximization of the mutual information:

\[
\max_{\mathbf{A}, \mathbf{B}} \log \frac{|\mathbf{I} + \mathbf{A}^H \mathbf{B} \mathbf{B}^H \mathbf{H}^H \mathbf{A}|}{|\mathbf{A}^H \mathbf{R}_n \mathbf{A}|}
\]
\[
\text{s.t.} \quad \text{Tr} (\mathbf{B}^H \mathbf{B}) \leq P_T
\]

The optimal solution diagonalizes the channel matrix and the MSE matrix.
Traditional Approach (IV)

- A diagonal structure implies:
  - scalarization of the problem (no matrices)
  - simplification of the problem (easy to solve)

- What happens with other design criteria ??
  - minimization of the maximum of the BER’s
  - minimization of the average BER
  - maximization of the geometric mean of the SINR’s

- Is the diagonal structure always optimal ??
Novel Approach: Unified Framework (I)

- General approach: design system by optimizing some arbitrary measure of quality of the system performance:
  - In terms of MSE:
    \[
    \begin{align*}
    \min_{\{A_k, B_k\}} & \quad f_0 \left( \left\{ \{\text{MSE}_{k,i}\}_{i=1}^{L}\right\}_{k=1}^{N} \right) \\
    \text{s.t.} & \quad \sum_{k=1}^{N} \text{Tr} \left( B_k^H B_k \right) \leq P_T
    \end{align*}
    \]
  - In terms of SINR:
    \[
    \begin{align*}
    \max_{\{A_k, B_k\}} & \quad f_0 \left( \left\{ \{\text{SINR}_{k,i}\}_{i=1}^{L}\right\}_{k=1}^{N} \right) \\
    \text{s.t.} & \quad \sum_{k=1}^{N} \text{Tr} \left( B_k^H B_k \right) \leq P_T
    \end{align*}
    \]
  - In terms of BER:
    \[
    \begin{align*}
    \min_{\{A_k, B_k\}} & \quad f_0 \left( \left\{ \{\text{BER}_{k,i}\}_{i=1}^{L}\right\}_{k=1}^{N} \right) \\
    \text{s.t.} & \quad \sum_{k=1}^{N} \text{Tr} \left( B_k^H B_k \right) \leq P_T
    \end{align*}
    \]
Novel Approach: Unified Framework (II)

- The optimum receiver is the Wiener filter
  \[ A_k = (H_k B_k B_k^H H_k^H + R_{n_k})^{-1} H_k B_k \]
  all the MSE’s, SINR’s, and BERs are simultaneously optimized.

- The MSE matrix reduces to
  \[ E_k = \left( I + B_k^H H_k^H R_{n_k}^{-1} H_k B_k \right)^{-1} \]

- The MSE’s, SINR’s, and BER’s can be expressed as a function of the diagonal elements of the MSE matrices:
  \[ \text{MSE}_{k,i} = [E_k]_{ii} \]
  \[ \text{SINR}_{k,i} = \frac{1}{\text{MSE}_{k,i}} - 1 \]
  \[ \text{BER}_{k,i} = \alpha_{k,i} Q \left( \sqrt{\beta_{k,i} \text{SINR}_{k,i}} \right) \]
Novel Approach: Unified Framework (III)

- Since the SINR’s and BER’s are directly related to the MSE’s, we can consider w.l.o.g. the problem formulation as a function of the MSE’s:

\[
\min_{\{B_k\}} f_0 \left( \left\{ \left\{ \left( \begin{bmatrix} 1 + B_k^H H_k^H R_{n_k}^{-1} H_k B_k \end{bmatrix}^{-1} \right) \right\}_{ii} \right\}_{i=1}^L \right)_{k=1}^N
\]

s.t. \( \sum_{k=1}^N \text{Tr} \left( B_k B_k^H \right) \leq P_T \)

- This is a terrible non-convex problem !!!

- Even in the scalar case, \( \frac{1}{1 + |b|^2 r} \) is non-convex.

- The problem needs to be simplified.
Solution to General Formulation

- **Theorem:** Consider the following constrained optimization problem

\[
\min_B f_0 \left( \left\{ \left( I + B^H H^H R_n^{-1} H B \right)^{-1} \right\}_{i=1}^L \right)
\]

s.t. \( \text{Tr} \left( B B^H \right) \leq P_T \)

- If \( f_0 \) is Schur-concave, then the optimal solution diagonalizes the channel and the MSE matrix \( B = U_{H,1} \Sigma_{B,1} \).

- If \( f_0 \) is Schur-convex, then the optimal solution diagonalizes the channel up to a rotation and the MSE matrix has equal diagonal elements (non-diagonal) \( B = U_{H,1} \Sigma_{B,1} V_B^H \).
Scalarized Problem (I)

- Original system

- Fully diagonalized system

- Diagonalized (up to a rotation) system
Schur-Concave Functions

- **Scalarized MSE:**

\[
\text{MSE}_i = \frac{1}{1 + z_i \lambda_{H,i}} \quad 1 \leq i \leq L
\]

- **Problem formulation**

\[
\min_{\{z_i\}} f_0 \left( \left\{ \frac{1}{1 + z_i \lambda_{H,i}} \right\}_{i=1}^{L} \right)
\]

s.t. \[
\sum_{i} z_i \leq P_T \quad z_i \geq 0
\]

- The solution depends on the specific choice of \( f_0 \).
Examples of Schur-Concave Functions

- Minimization of the (weighted) sum of the MSE’s.
- Minimization of the (weighted) product of the MSE’s.
- Minimization of the determinant of the MSE matrix.
- Maximization of the mutual information.
- Maximization of the (weighted) sum of the SINR’s.
- Maximization of the (weighted) product of the SINR’s.
- Minimization of the product of the BER’s.
Schur-Convex Functions

- Scalarized MSE:
  \[ \text{MSE}_i = \frac{1}{L} \sum_{j=1}^{L} \frac{1}{1 + z_i \lambda_{H,j}} \quad 1 \leq i \leq L \]

- Problem formulation
  \[
  \min_{\{z_i\}} \quad \begin{cases} 
  f_0 \left( \left\{ \frac{1}{L} \sum_{j=1}^{L} \frac{1}{1 + z_j \lambda_{H,j}} \right\}_{i=1}^{L} \right) \\
  \text{s.t.} \quad \sum_i z_i \leq P_T \\
  z_i \geq 0 
  \end{cases}
  \]

  which reduces to (independent of \( f_0 \))
  \[
  \min_{\{z_i\}} \quad \sum_i \frac{1}{1 + z_i \lambda_{H,i}} \quad \begin{cases} 
  \text{s.t.} \quad \sum_i z_i \leq P_T \\
  z_i \geq 0 
  \end{cases}
  \]

  with solution given by the water-filling (plus the rotation)
  \[
  z_i = \left( \mu \lambda_{H,i}^{-1/2} - \lambda_{H,i}^{-1} \right)^+ 
  \]
Examples of Schur-Convex Functions

- Minimization of the maximum of the MSE’s.
- Maximization of the minimum of the SINR’s.
- Maximization of the harmonic mean of the SINR’s.
- Minimization of the average BER.
- Minimization of the maximum of the BER’s.
Simulations (I)

Scenario

- Parameters of the HIPERLAN/2 European standard for WLAN.

- Frequency-selective multi-antenna wireless MIMO channel:
  - Frequency-selectivity: power delay profile type C for HIPERLAN/2.
  - Spatial correlation as measured in real scenarios (no i.i.d. matrix elements).

- Perfect CSI at both sides of the link.
Simulations (II)

Power-Constrained System

- BER (QPSK) vs. SNR
- 4x4 MIMO, L=2
- SUM-BER: best performance
- MAX-BER: 2nd best performance
- HARM-SINR: 3rd best performance
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QoS-Constrained Systems

- Previous approach: maximization of the quality of the system subject to a Tx-power constraint

\[
\min_{\{A_k, B_k\}} \quad f_0 \left( \left\{ \{\text{MSE}_{k,i}\}_{i=1}^{L} \right\}_{k=1}^{N} \right)
\]
\[
\text{s.t.} \quad \sum_{k=1}^{N} \text{Tr} \left( B_k^H B_k \right) \leq P_T
\]

- It may be useful to consider the opposite formulation

\[
\min_{\{A_k, B_k\}} \quad \sum_{k=1}^{N} \text{Tr} \left( B_k^H B_k \right)
\]
\[
\text{s.t.} \quad f_0 \left( \left\{ \{\text{MSE}_{k,i}\}_{i=1}^{L} \right\}_{k=1}^{N} \right) \leq \alpha
\]

- The same results hold for both problems.

- From system level viewpoint: independent QoS constraints.

- This approach has been extensively considered in multiuser systems.
QoS-Constrained Problem Formulation (I)

- Design system by minimizing the transmit power subject to a set of QoS constraints:
  
  - In terms of MSE:
    \[
    \begin{align*}
    \min_{\{A_k, B_k\}} & \quad \sum_{k=1}^{N} \text{Tr} \left( B_k^H B_k \right) \\
    \text{s.t.} & \quad \text{MSE}_{k,i} \leq \rho_{k,i} \quad 1 \leq i \leq L, \ 1 \leq k \leq N
    \end{align*}
    \]

  - In terms of SINR:
    \[
    \begin{align*}
    \min_{\{A_k, B_k\}} & \quad \sum_{k=1}^{N} \text{Tr} \left( B_k^H B_k \right) \\
    \text{s.t.} & \quad \text{SINR}_{k,i} \geq \gamma_{k,i} \quad 1 \leq i \leq L, \ 1 \leq k \leq N
    \end{align*}
    \]

  - In terms of BER:
    \[
    \begin{align*}
    \min_{\{A_k, B_k\}} & \quad \sum_{k=1}^{N} \text{Tr} \left( B_k^H B_k \right) \\
    \text{s.t.} & \quad \text{BER}_{k,i} \leq p_{k,i} \quad 1 \leq i \leq L, \ 1 \leq k \leq N
    \end{align*}
    \]
QoS-Constrained Problem Formulation (II)

- The optimum receiver is again the Wiener filter.
- The SINR’s and BER’s are directly related to the MSE’s.
- We can consider w.l.o.g. the problem formulation with QoS constraints in terms of MSE’s:

$$\min_{\{B_k\}} \sum_{k=1}^{N} \text{Tr} \left( B_k B_k^H \right)$$

s.t. $$\left[ \left( I + B_k^H H_k^H R_{n_k}^{-1} H_k B_k \right)^{-1} \right]_{ii} \leq \rho_{k,i} \quad 1 \leq i \leq L, \ 1 \leq k \leq N$$

- Highly non-convex problem needs to be simplified.
- Using majorization theory, we can reformulate it as a simple convex problem that can be optimally solved.
Suboptimum Solution

- Impose a diagonal structure $\mathbf{B} = \mathbf{U}_{H,1} \mathbf{\Sigma}_{B,1}$.

- The problem simplifies to

\[
\min_{\mathbf{\Sigma}_B} \quad \text{Tr}(\mathbf{\Sigma}_B \mathbf{\Sigma}_B^H) \\
\text{s.t.} \quad \left[ \left( \mathbf{I} + \mathbf{\Sigma}_B^H \mathbf{D}_H \mathbf{\Sigma}_B \right)^{-1} \right]_{ii} \leq \rho_i \quad 1 \leq i \leq L
\]

\[
\downarrow \quad z_i \triangleq |[\mathbf{\Sigma}_B]_{ii}|^2
\]

\[
\min_{\{z_i\}} \quad \sum_{i=1}^{L} z_i \\
\text{s.t.} \quad \frac{1}{1+z_i \lambda_{H,i}} \leq \rho_i \quad 1 \leq i \leq L \quad \iff \quad z_i = \lambda_{H,i}^{-1} (\rho_i^{-1} - 1)
\]
Optimum Solution

- **Theorem:** The solution to the following constrained non-convex optimization problem

\[
\begin{align*}
\min_B & \quad \text{Tr}(BB^H) \\
\text{s.t.} & \quad \left[ \left( I + B^H H R_n^{-1} H B \right)^{-1} \right]_{ii} \leq \rho_i \quad 1 \leq i \leq L
\end{align*}
\]

is \( B = U_{H,1} \Sigma_{B,1} V_H^H \) where \( z_i \triangleq |[\Sigma_{B,1}]_{ii}|^2 \) is the solution to the simple convex optimization problem

\[
\begin{align*}
\min_{\{z_i\}} & \quad \sum_{i=1}^L z_i \\
\text{s.t.} & \quad \sum_{i=k}^L \frac{1}{1+z_i \lambda_{H,i}} \leq \sum_{i=k}^L \rho_i \quad 1 \leq k \leq L \\
z_k & \geq 0
\end{align*}
\]

which has a multi-level water-filling solution

\[
z_i = \left( \mu_i^{1/2} \lambda_i^{-1/2} - \lambda_i^{-1} \right)^+
\]
Simulations

QoS-Constrained System

- Tx-power vs. BER
- 4x4 MIMO, L=3
- QoS constraints in terms of BER
- 2-3dB gain with the optimum solution
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➡️ • Robust Design against Channel Estimation Errors

• Conclusions
Robust Design (I)

- Previous approaches considered perfect CSI.
- In real systems, CSI is imperfect since it has to be estimated.
- Robust designs are necessary in practice.
Robust Design (II)

- Two main philosophies to introduce robustness:
  - Worst-case approach (pessimistic)
    \[ H = \hat{H} + H_\Delta \quad \|H_\Delta\| \leq \epsilon_H \]
    \[ R_n = \hat{R}_n + R_{n,\Delta} \quad \|R_{n,\Delta}\| \leq \epsilon_n. \]
  - Stochastic (Bayesian) approach
    \[ \mathbb{E}_{H,R_n|\hat{H},\hat{R}_n}[\cdot] \]
Worst-Case Robust Design

- **Power-constrained system**

\[
\begin{align*}
\min_{A,B} \quad & \max_{H,R_n} f_0 \left( \{\text{MSE}_i\}_{i=1}^{L} \right) \\
\text{s.t.} \quad & \text{Tr} \left( BB^H \right) \leq P_T, \\
& H = \hat{H} + H_\Delta \\
& R_n = \hat{R}_n + R_{n,\Delta}
\end{align*}
\]

\( H_\Delta : \|H_\Delta\| \leq \epsilon_H, \quad R_{n,\Delta} : \|R_{n,\Delta}\| \leq \epsilon_n, \quad R_n = R_n^H > 0 \)

- **QoS-constrained system**

\[
\begin{align*}
\min_{A,B} \quad & \text{Tr} \left( BB^H \right) \\
\text{s.t.} \quad & \max_{H,R_n} \text{MSE}_i \leq \rho_i, \quad 1 \leq i \leq L, \\
& H = \hat{H} + H_\Delta \\
& R_n = \hat{R}_n + R_{n,\Delta}
\end{align*}
\]

\( H_\Delta : \|H_\Delta\| \leq \epsilon_H, \quad R_{n,\Delta} : \|R_{n,\Delta}\| \leq \epsilon_n, \quad R_n = R_n^H > 0 \)
Stochastic Robust Design

- Power-constrained system

\[
\begin{align*}
\min_{A, B} \quad & \mathbb{E}_{H, R_n | \hat{H}, \hat{R}_n} f_0 \left( \{\text{MSE}_i\}_{i=1}^L \right) \\
\text{s.t.} \quad & \text{Tr} \left( BB^H \right) \leq P_T
\end{align*}
\]

- QoS-constrained system

\[
\begin{align*}
\min_{A, B} \quad & \text{Tr} \left( BB^H \right) \\
\text{s.t.} \quad & \mathbb{E}_{H, R_n | \hat{H}, \hat{R}_n} \text{MSE}_i \leq \rho_i, \quad 1 \leq i \leq L
\end{align*}
\]
Simulations

- Average and worst-case MSE vs. desired MSE.
- 4x4 MIMO, L=3
- Stochastic design satisfies QoS on average (not on the worst-case).
- Worst-case design satisfies QoS on the worst-case (on average is over-designed).
- Worst-case design requires a huge increase of Tx-power $\Rightarrow$ impractical.
Outline of Presentation

• Motivation

• MIMO Channels: An Overview

• Capacity of MIMO Channels

• Joint Tx-Rx Design for MIMO Channels
  – Power-Constrained Systems: A Unified Framework
  – QoS-Constrained Systems

• Robust Design against Channel Estimation Errors

⇒ • Conclusions
Conclusions

• When the channel uncertainty at the Tx is sufficiently high $\Rightarrow$ uniform power allocation.

• Joint Tx-Rx design for MIMO channels $\Rightarrow$ unified framework that embraces previous results and generalizes upon them (Schur-convexity):
  - Formulation within the general and powerful framework of convex optimization theory.
  - Optimum and practical water-filling algorithms for a variety of design criteria (min MAX-BER).
  - Closed-form solution for the minimum AVE-BER.

• Optimum solution for the QoS-constrained design $\Rightarrow$ practical water-filling algorithm.
- End of Presentation -
Future Research Lines

- Capacity analysis of MIMO channels
  - Iterative-waterfilling for the whole capacity region.
  - Iterative-waterfilling for beamforming systems.
  - Characterization of the outage capacity in convex form

- Joint Tx-Rx design for MIMO channels
  - Extend the unified framework based on Schur-convexity to include a DF scheme and/or peak constraints.
  - Deep treatment of the robust design.
  - Generalization of the results to the multiuser case.
Preliminary: Single-Link Performance

- The performance of a link is commonly measured in terms of MSE, SINR, or BER:

\[
\text{MSE} \triangleq \mathbb{E} \left[ |\hat{x} - x|^2 \right] = |a^H b - 1|^2 + a^H R_{in} a
\]

\[
\text{SINR} \triangleq \frac{|\text{desired component}|^2}{|\text{interference-plus-noise component}|^2} = \frac{|a^H b|^2}{a^H R_{in} a}
\]

\[
\text{BER} \triangleq \frac{\# \text{ bits in error}}{\text{total } \# \text{ bits}} = \alpha Q\left(\sqrt{\beta \text{ SINR}}\right)
\]
Preliminary: Wiener Filter

• The optimum linear receiver is the Wiener filter:
  
  \[ a_k = (H_k b_k b_k^H H_k^H + R_{n_k})^{-1} H_k b_k \]

  – Single beamforming:

  \[ a_k = (H_k b_k b_k^H H_k^H + R_{n_k})^{-1} H_k b_k \]

  – Multiple beamforming:

  \[ A_k = (H_k B_k B_k^H H_k^H + R_{n_k})^{-1} H_k B_k \]

• It is optimum in the sense that
  
  – each of the MSE’s is simultaneously minimized,
  
  – each of the SINR’s is simultaneously maximized, and
  
  – each of the BER’s is simultaneously minimized.