BLIND EQUALIZATION OF CDMA SYSTEMS USING SPATIAL AND TEMPORAL DIVERSITY RECEIVERS

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ABSTRACT

The paper addresses the blind equalization problem of spread spectrum modulations in the presence of fast time-variant frequency selective channels. The basic assumption of the paper is that the channel response exhibit fast changes. A second goal of the paper is to force the definition of a universal CDMA blind equalization scheme. The formulation of the equalization scheme allows the consideration of temporal and/or spatial diversity front-end receivers. The result is a high performance system that uses a deterministic blind criterion to equalize the channel avoiding the use of stochastic methods. The proposed technique performs direct channel equalization without previous channel estimation. It is also shown how the proposed method can perform equalization working at lower than chip rates, in special interest to reduce the equalization speed.

1. INTRODUCTION

Spread spectrum technology is in special interest when dealing with wireless fading channels because it has its own capabilities to combat the hostile channel effect. However due to the frequency selectivity of the channel, the orthogonality between user codes at the receiver is lost, and some algorithms to combat this channel impact must be approached.

The RAKE correlator is the optimum receiver for processing a spread spectrum signal, but it requires estimating the channel coefficients, which is not feasible if the channel is changing rapidly. Another algorithms are based in correlation techniques, but assume that the channel parameters change slowly considering a linear time invariant system in sufficient short intervals [1]. Recent studies [2] and references, use a constrained optimization of the receiver’s output variance as a simple method for designing the blind multiuser receiver, but still use the covariance matrix. The best solution considering the time-variant nature of this kind of channels requires the use of deterministic methods, which make use of the a priori known signal structure, avoiding the use of conventional stochastic algorithms.

This paper is the natural extension of works done in [3] and [4], where equalizers based on a deterministic criterion were developed to directly identify the transmitted signal by means of spatial and/or temporal diversity. It proposes a mobile communications scheme that combines the use of CDMA systems with a deterministic blind equalizer. The result is a system that exploits the inherent structure of the transmitted data due to the spreading process, obtaining a robust equalization criterion against the frequency selectivity of the fading channel.

In special interest are spread spectrum equalizers that perform equalization at symbol rate. The main advantage of those equalizers is that they do not work at high chip rates, a drawback when using long spreading code sequences. A design achieving equalization at lower rates is presented in [5], and applied in a single user DS-CDMA scheme. The current paper also suggests how to take into account the solution proposed in [5] reducing the equalization speed at lower than chip rates in a multiuser scheme.

Next section summarizes briefly the signal model in a multiuser CDMA system. Section 3 justifies equalization design using the criterion suggested in [3] and [4], and obtains the proposed channel equalization algorithm. Single user and multiuser cases are separately studied. To conclude that section a solution to achieve equalization at rates lower than chip rate is proposed. Section 4 derives the CRB covariance matrix for the transmitted symbol estimator in order to study the performance of the algorithm. Finally section 5 presents some simulations results to illustrate the proposed scheme.

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2. SIGNAL MODEL

In a multiuser CDMA system several users can transmit simultaneously, and the received signal is the superposition of the signals from all users plus additive zero-mean, white, Gaussian noise \( n(t) \) with variance \( \sigma_n^2 \):

\[
r(t) = \sum_{i=1}^{K} r^i(t) + n(t)
\]

Let us define \( r^k(t) \) as the \( k \) user received signal expressed as the convolution of the transmitted signal with the time-variant frequency selective channel with baseband impulse response \( h(\tau t) \).

\[
r^k(t) = h^k(\tau t) * s^k(t)
\]

Finally the transmitted signal for \( k \) user \( s^k(t) \) is given by:

\[
s^k(t) = \sum_{i=0}^{N_t-1} d^k_i \sum_{i=0}^{L_c-1} c^k_i p(t - iT_c - nT_s)
\]

where \( d^k_i \in \{ \pm 1 \pm j \} \) are the \( k \) transmitted bits, \( c^k_i \) is the PN sequence, \( L_c \) is the length of the spreading code, \( p(t) \) is the chip pulse shaping, and \( T_c \) is the chip period.

3. PROBLEM STATEMENT

Writing the model in (3) as a matrix model, the multiuser DS-CDMA frequency selective fading channels equalization problem can be identified with the equalization criterion proposed in [4], which is based on the insertion of certain structure in the transmitted information using a linear transformation. The proposed resulting algorithm is based on the assumption that the \( B \) channels have no common zeros, the perfect channel equalization is allowed [3].

To simplify the notation let us first obtain the expressions for the single user model, and later we will generate the multiuser scheme.

A. Single user

If \( d \) is a vector that contains the \( N_t \) information symbols:

\[
d = [d_0 \ d_1 \ ... \ d_{N_t-1}]^T
\]

considering a rectangular pulse for \( p(t) \), and sampling the signal at \( Ts = T_c \) (one sample per chip), the spreading can be modeled by means of a matrix as:

\[
G_c = \begin{bmatrix} c & 0_{L_c} & ... & 0_{L_c} \\ 0_{L_c} & c & ... & 0_{L_c} \\ ... & ... & ... & ... \\ 0_{L_c} & 0_{L_c} & ... & c \end{bmatrix}
\]

where \( \theta_{L_c} \) is a \( L_c \times 1 \) zero vector, and \( c \) is the spreading code

\[
c = [c_0 \ c_1 \ ... \ c_{L_c-1}]^T
\]

From the above definitions the spread sequence \( s \) can be seen as the result of a linear transformation by means of the spreading matrix \( G_c \) over the transmitted data \( d \):

\[
s = G_c d
\]

The spreading matrix \( G_c \) is a \( (G_c \times N_t) \times N_s \) column full rank matrix that provides structure to the encoded vector \( s \). The transformation defines a signal subspace \( S \) spanned by the \( N_t \) columns of matrix \( G_c \), where vector \( s \) is contained.

Accordingly it is possible to define an orthogonal subspace \( S^\perp \) spanned by an \( r = (Gc-1) \times N_u \) dimensional orthogonal basis and obtain its associated check matrix \( G_c^\perp \). By definition this matrix accomplishes:

\[
G_c^\perp G_c d = G_c^\perp s = 0
\]

The check matrix will detect changes between the transmitted code and the received information. The outputs of the marginal channels \( C^i \) \( (i=1,2...B) \) to the transmitted data \( s \), and the channel noise contributions \( \eta \) will force the received data to be contained in \( S \oplus S^\perp \). Basically, the projection of the received data in the orthogonal subspace \( S^\perp \) is used by the equalizer to characterize the noise and channel response.

Thus, the two equalizer design equations becomes [4]:

\[
\hat{s} = Y_t e
\]

\[
G_c^\perp \hat{s} = G_c^\perp Y_t e = 0
\]

In (9) \( e \) is the vector that contains the equalizer coefficients, and \( Y_t \) are the first \( n \) rows of the convolution matrix \( Y \) that contains the symbols at the output of the channel.

\[
\begin{bmatrix} y_{11} \\ y_{12} \\ y_{13} \end{bmatrix} = \begin{bmatrix} y_{11} \\ y_{12} \\ y_{13} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}
\]

It is clear from (9) that an estimation of the transmitted vector \( s \) is obtained in the first equation by means of the equalizer \( e \). Examining the second equation we can conclude that it contains information on the residual ISI and noise in the orthogonal signal subspace \( S^\perp \), and might be zero if a perfect equalization is accomplished.

The previous discussion reveals that the equalizer coefficients can be obtained by means of a cost function that maximizes the signal to noise+ISI ratio in the orthogonal subspace (SNIR):

\[
SINR = \frac{E[H^HY_t Y_te]}{E[H^HY_t Y_c^\perp]}
\]

where \( G_c^\perp \) is the spreading code for the orthogonal subspace.
and the equalizer that maximizes (10) corresponds to the
generalized eigenvector associated with the maximum
generalized eigenvalue:
\[ Y^H_G Y e = \lambda_{max} Y^H G c_j^* Y e \]  \hspace{1cm} (11)

B. Multiuser
Defining \( d^j \) as the symbol vector (eq. 4) and \( G_c^k \) the spreading matrix (eq. 5) for user \( k \), the spreaded sequence can be modeled in a matrix notation as:
\[ s = s^I + s^2 = \begin{bmatrix} G_c^1 & G_c^2 \\ d^1 & d^2 \end{bmatrix} \]  \hspace{1cm} (12)
considering without lost of generality the 2 users case.

In the multiuser case the spreading matrix is a column full rank matrix \( (G_c x N_s)xK \). Hence the equalization scheme can be used again.

Let us now study how to ensure the perfect equalization criterion [4] in the multiuser case the required redundancy (i.e. the minimum number of rows in \( G_c \)) must be:
\[ \text{Redundancy} = \text{rank}(G_c^k S_\nu) \geq K(L + v - 2) \]  \hspace{1cm} (13)

Proof: Focusing on \( k \) user let us denote \( C^{k,i} \) as the Sylvester matrix that contains the \( i \)-channel coefficients, where \( i \) is the diversity branch \( i = 1 \ldots B \). Thus (8) can be written as:
\[ y^I = \begin{bmatrix} C_{11} & \ldots & C_{1B} \\ \vdots & \ddots & \vdots \\ C_{K1} & \ldots & C_{KB} \end{bmatrix} \begin{bmatrix} s^1 \\ \vdots \\ s^K \end{bmatrix} \]  \hspace{1cm} (14)
where \( y^I \) is the vector at the output of the \( i \)-channel.

Combining all branches, the equalizer output is given by:
\[ r = \begin{bmatrix} s^I \\ \vdots \\ s^K \end{bmatrix} \begin{bmatrix} C_{11} & \ldots & C_{1B} \\ \vdots & \ddots & \vdots \\ C_{K1} & \ldots & C_{KB} \end{bmatrix} \begin{bmatrix} e^1 \\ \vdots \\ e^B \end{bmatrix} \]  \hspace{1cm} (15)
where \( S' \) is the Sylvester matrix whose columns are shifted versions of the \( k \)-user transmitted samples \( \tilde{s}^k \).

Finally to design the equalizer coefficients satisfying the second equation in (9) means:
\[ G_c^k Y e = G_c^k S_t C e \]
\[ = \begin{bmatrix} 0 & p_2^1 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ p_2^L & \ldots & 0 \end{bmatrix} C e = 0 \]  \hspace{1cm} (16)
where \( S_t = S(I:n,:) \) is by definition a sub-matrix of \( S \) matrix composed by rows \( I \) to \( n \), such that for each user the first columns is the transmitted vector \( \tilde{s}^k \) (orthogonal to \( G_c^c \)).

If the column vectors \( \{p_2^j\}_{j=2}^{L+v-1} \) are linearly independent, the solution for equation (16) is equivalent to the perfect equalization for each user:
\[ Ce = [\beta^1 \ 0 \ \ldots \ \beta^2 \ 0 \ \ldots \ \beta^K] \]  \hspace{1cm} (17)
where \( \beta^k \) is an arbitrary multiplicative complex constant for user \( k \). Thus at the output of the equalizer:
\[ r = \sum_{i=1}^{K} \beta^i s^i \]  \hspace{1cm} (18)
Forcing the column vectors in (16) to be linearly independent means that the minimum required redundancy for perfect channel equalization is given by expression (13).

Unfortunately temporal diversity receivers need to oversample the received signal at \( B \) samples per chip, increasing the number channel coefficients, and forcing the equalizer to work at high rates. As proposed in [5] it is possible to perform the blind channel equalization working at lower than chip rates.

As depicted in figure 1, processing the despreading at the output of the channel and before the equalization, the signal is oversampled at \( B \cdot S \cdot G_s \) samples per symbol and thus an equalizer exploiting this kind of temporal diversity can be used. As analyzed in [5] the equalizer works over the equivalent channel that includes spreading, frequency-selective channel distortion and despreading processes.

\[ x(t) \rightarrow b(t) \rightarrow w(t) \rightarrow h_{eq}(t) \rightarrow r(t) \rightarrow \text{Equalizer} \rightarrow x'(t) \]

Figure 1. General scheme

Notice that in the proposed multiuser receiver, it is not possible to carry out despreading before equalization. That is because it assumes the information vector at the output of the equalizer to be still the spreaded information vector.

This result may seem discouraging. However if the spreading process is carry out in two steps, it will be possible to consider the despreading before the equalizer, and still accomplish (9) at the equalizer output.

Figure 2 illustrates how the spreading sequence is achieved. The first spreading code provides the structure to the transmitted data, and can be used to perform the equalization (6). The second spreading code is used to achieve the despreading at the output of the channel (fig.1).

Information Data
Rate T
1st Spreading Code
Rate T_s
2nd Spreading Code
Rate T_e

Figure 2. Two steps spreading process
4. PERFORMANCE ANALYSIS

In the sequel, we derive the Cramér-Rao Bound (CRB) for the transmitted symbol estimator for user \( k \), obtaining a lower bound that will allow us to evaluate the mean-square error (MSE) of the proposed scheme.

Following the same notation used in previous sections, \( y' \) is the vector at the output of the channel for the branch \( i = 1...B \). And focusing on user \( k \), \( C^k \) is the matrix that contains the \( i \) channel coefficients, and \( s^k \) corresponds to its transmitted vector:

\[
y' = \begin{bmatrix}
    c^{k1} \\
    ... \\
    c^{kL}
\end{bmatrix} s^k
\]

(19)

The likelihood function of the data is given by:

\[
f(y;s) = \frac{1}{(2\pi \sigma^2_w)^B} \exp \left\{ -\frac{1}{\sigma^2_w} \sum_{i=1}^{B} (y' - C^i s)^H (y' - C^i s) \right\}
\]

(20)

And denoting \( A \) as:

\[
A = \frac{\partial}{\partial s} \ln[f(y;s)] = \frac{1}{\sigma^2_w} \sum_{i=1}^{B} C^k H (y' - C^i s)
\]

(21)

the Fischer information matrix (FIM) for the symbol vector \( s^k \) is given by:

\[
J = E\{AA^H\} = \left( \frac{1}{\sigma^2_w} \right)^2 \sum_{i,j=1}^{B} E\{C^k H (y' - C^i s)(y' - C^j s)^H C^k\}
\]

(22)

The expression \( (y' - C^i s) \) is the noise vector \( w' \), and computing the expectation:

\[
E\{w'w'^H\} = \sigma^2_w I \delta(i-j)
\]

(23)

we obtain:

\[
J = \frac{1}{\sigma^2_w} \sum_{i=1}^{B} C^k H C^k
\]

(24)

And thus the CRB can be obtained directly from the FIM computing its inverse:

\[
CRB = J^{-1} = \sigma^2_w \left( C^k H C^k \right)^{-1}
\]

(25)

where the matrix \( C^k \) is defined as:

\[
C^k = \begin{bmatrix}
    C^{k1} \\
    ... \\
    C^{kB}
\end{bmatrix}^T
\]

(26)

5. SIMULATIONS

To illustrate the performance of the proposed algorithm it has been tested and compared with the analytical MSE expression obtained in section 4.

In all cases the length of the transmitted frame information was 32 QPSK data symbols, the channel EbNo was 12dB and the FIR filters in each equalizer branch had 8 coefficients. The order of the spatial and temporal diversity was \( B=6 \). Finally the PN sequences considered were Gold codes of length 15 chips.

The frequency selective multipath channel responses were generated according to:

\[
h(n) = \sum_{i=1}^{L} \alpha_i \delta(n-i)
\]

(27)

where \( h(n) \) is the channel response, \( L \) the length of the channel, in our case the channel length was five chips, and \( \alpha_i \) the random complex value for path \( i \).

Figure 3 shows the performance of the spatial diversity receiver; while figure 4 illustrates the temporal diversity one. In both cases simulations display the percentage of realizations (over 1000) for which the equalizer output EbNo was higher than the value indicated in the x-axis. The results are focused on user 1 and several simulations were done increasing the number of users (maintaining the same channel for user 1). As it can be seen when the number of users increases the performance of the algorithm decreases because the number of channels to equalize is higher.

![Figure 3. Algorithm Performance in Spatial Diversity. EbNos=12dB. No. users:1-4](image)

Notice that in the spatial diversity receiver, \( B \) sensors are considered, and so the maximum output EbNo could be:

\[
\max\{(EbNo)_{out}\} = (EbNo)_{in} + 10 \log(B)
\]

(28)
As presented at the end of section 3, Figure 5 illustrates the equalization process working at lower than chip rates. A 32 chip PN sequence was used. The spreading process was done in two steps. The 1st spreading code was a 8 chip PN sequence; based on 7 length Gold sequence, plus an additional chip (±1). The 2nd spreading code was a 4 chip PN sequence.

Figure 6 compares the CRB derived in section IV with the normalized root-mean square error (RMSE) defined bellow:

\[
RMSE = \sqrt{\frac{1}{N_r} \sum_{i=1}^{N_r} \left| d_i - \hat{d}_i \right|^2}
\]

(29)

where \( N_r \) is the number of Monte Carlo realizations (500). The RMSE is employed as a performance measure for the proposed equalizer based on spatial and temporal diversity receivers.

It is seen that the RMSE decreases as \( 1/\text{EbNo} \), as CRB does.

6. CONCLUSIONS

A blind equalization scheme for CDMA systems in multipath frequency selective fading channels has been introduced. The main contribution in this work is the exploitation of the inherent structure in the transmitted data due to orthogonality properties of the spreading codes. The spreading process allows defining an orthogonal subspace \( S^\perp \), which is used in the receiver to obtain a robust equalization criterion against the frequency selective fading channels.

The resulting equalizer, applied over single user and multiuser CDMA systems, exploit the high capacity of blind algorithms based on block coded modulations avoiding the insertion of certain redundancy in the transmitter. Furthermore, equalization algorithms working at lower than chip rates is proposed and simulated.

7. REFERENCES


