ANTENNA ARRAYS

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Abstract

The reduction of the errors in the observables caused by the multipath propagation and the interferences is addressed. Two different estimators are derived by applying the maximum likelihood (ML) principle to a signal model that assumes the reception of several reflections of the GNSS signal and that the spatial signatures of all the signals are arbitrary and unstructured. The first estimator, which is a classical result since the noise is modeled as spatially white, is an extension of the well-known multipath estimating delay-lock-loop. The second estimator is derived by assuming that the spatial correlation matrix of the noise is unknown. This endows the estimator with interference cancelation capability, of which the first estimator lacks. The second method constitutes a new result and its performance is always equal or better than that of the first one. Moreover, we propose an approximation of the estimator for the correlated-noise case that provides the same performance as the original criterion. This approximation may allow the use of a computationally simple optimization algorithm that was only applicable in the white-noise case.

1 Introduction

The fast evolution of the applications of GNSS systems is leading to increasingly stringent requirements for their performance, particularly in regard to their accuracy. It is recognized that neither GPS nor GLONASS can fulfill the requirements of the more demanding applications. This is the reason why augmentation programs for the existing systems have been set up, and the development of a new global navigation satellite system, GALILEO, is being planned [1]. The innovations in the space and control segments will reduce several types of errors, but the receptor remains as a fundamental element in obtaining an overall satisfactory performance. Therefore, a research effort need to be done in order to design reception schemes that are able to take advantage of the new augmented systems. To this end, the GNSS receivers should combat the disturbing effects of the interferences and the multipath propagation; and this the issue on which this paper focuses.

Multipath propagation and interference can be considered as the dominant error sources in most high precision applications and are the limiting factors in achieving the ultimate GNSS accuracy [2]. Coherent multipath degrades the pseudorange and carrier phase measurements because the synchronization algorithms in conventional receivers lock to a combination of the direct or line-of-sight signal (LOS) with the coherent reflections. It is especially serious the case of static multipath, which causes constant biases of the pseudorange and carrier phase measurements. Moreover, external interference may increase the errors in the observables by several magnitude orders, so receivers should be endowed with interference canceling techniques in order to make possible the use of GNSS in safety-critical applications.

Due to the limitations of the multipath and interference mitigating techniques proposed to date (see [2], [3] and references therein) we find necessary to further investigate new methods to combat those error sources. In [3] it was shown that the use of an antenna array in the receiver is a very efficient alternative. In that work the knowledge of the direction-of-arrival (DOA) of the direct signal was exploited. On the other hand, in this paper we present two criteria that do not require that knowledge, but they assume that the spatial signatures of all the signals are arbitrary and unknown.

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Let consider that an $m$ element antenna array receives $d$ replicas of the signal transmitted by a GNSS satellite. The signal with the smallest delay is the LOSS, and it will be denoted by the subscript 0. The other replicas are the reflections originated by nearby objects or surfaces. Then, the received complex signal at the $l$th antenna is

$$x_l(t) = \sum_{k=0}^{d-1} \alpha_{l,k} q(t - \tau_k) + w_l(t) \quad l = 1 \ldots m$$  \hspace{1cm} (1)$$

where $\alpha_{l,k}$ is the complex amplitude of the $k$th replica, $\tau_k$ is the time-delay or pseudorange of the corresponding signal and $w_l(t)$ models the thermal noise and all other interferences. Note that a Doppler-frequency shift has not been introduced in (1). This amounts to assuming that the Doppler has been already acquired and that the differences between the frequency shifts of all the $d$ signals is much smaller than the observation interval employed in the estimators proposed herein. The direct-sequence spread-spectrum (DS-SS) signal $q(t)$ transmitted by the satellite can be expressed as:

$$q(t) = \sum_{m=-\infty}^{\infty} d(m) p(t - mT) \quad p(t) = \sum_{i=0}^{N-1} c(i) s(t - iT_c)$$  \hspace{1cm} (2)$$

where $d(m)$ are the symbols of the navigation message transmitted at rate $1/T$, $s(t)$ is the chip-shaping pulse, $T_c$ is the chip period and $c(i)$ represents the chips of the pseudo-noise (PN) code, whose length is $N = T/T_c$. This model of the received signals is generic, and specific values can be assigned to the parameters in order to represent the existing GNSS systems as well as the future systems presently under study. For clarity in the notation, let assume that the received signals are sampled twice per chip. Nevertheless, the model below can be extended to any sampling frequency equal to a rational fraction of the chip rate. We consider that the length of $s(t)$ is at most $LT_c/2$, with $L$ odd. Therefore, the $2N + L - 2$ samples of $q(t - \tau)$ taken during the interval that comprises essentially the $j$th symbol can be arranged as follows:

$$\begin{bmatrix} q(jT - (L - 1)T_c/4) & \ldots & q((j + 1)T + (L - 5)T_c/4) \end{bmatrix} = d(j) s^T(\tau) C + d(j-1)\ldots + d(j+1)\ldots$$  \hspace{1cm} (3)$$

where the $L \times 1$ vector $s(\tau)$ contains shifted samples of the shaping pulse and the matrix $C$ is formed with the chips of the PN code:

$$s^T(\tau) = \begin{bmatrix} s(-(L-1)T_c/4-\tau) & s(-(L-3)T_c/4-\tau) & \ldots & s((L-1)T_c/4-\tau) \end{bmatrix}$$  \hspace{1cm} (4)$$

$$C = \begin{bmatrix} c(0) & 0 & c(1) & 0 & c(2) & \ldots & c(N-1) & \ldots & 0 \\
0 & c(0) & 0 & c(1) & 0 & \ldots & 0 & c(N-1) & \ldots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & c(N-4) & 0 \\
c(N-4) & c(N-3) & 0 & c(N-2) & 0 & \ldots & \ldots & \ldots & \ldots \end{bmatrix}$$  \hspace{1cm} (5)$$

In (3) only the contribution of the central symbol appears explicitly, because the terms corresponding to the adjacent symbols are very small when previous coarse synchronization has been achieved. Using (1) and (3), the samples received by the antenna array during the $j$th interval can be written as

$$X_j = d(j) \ A S(\tau) C + V_j$$  \hspace{1cm} (6)$$

where

$$S(\tau) = \begin{bmatrix} s(\tau_0) & s(\tau_1) & \ldots & s(\tau_{d-1}) \end{bmatrix}^T$$  \hspace{1cm} (7)$$

and $\tau = [\tau_1 \ldots \tau_{d-1}]^T$. Each column of the $m \times (2N + L - 2)$ matrix $X_j$ contains the samples received in all the antennas in a given sampling instant. The matrix $V_j$ is formed identically to $X_j$ and includes the contribution of the “noise” $w_l(t)$ and the “interfering” symbols in (3). The $l$, $k$th entry

$^{1}(r)^T$, $(r)^H$ denote transpose and conjugate transpose, respectively.
of matrix $A$ is $e_{j,k}$, so this matrix contains the spatial signatures of the sources. We do not use an explicit parameterization of $A$ in terms of the DOAs of the sources, instead we treat its elements as deterministic parameters to be estimated. Thus, we avoid the need of precise information about the array calibration. It is worth mentioning here that the phase of the elements in the first column of $A$ is the carrier phase observable, in each antenna. Hence, the estimation of this matrix is also of primary interest in a GNSS receiver. Since many effects contribute to $V_j$, we model each of its columns as a circularly symmetric Gaussian vector which is temporally white and has an arbitrary spatial correlation matrix $Q$. If the noise is temporally correlated, all the following estimators can still be applied, but they will no longer be ML estimators.

The model presented above allows to formulate the despreading operation, performed in any DS-SS receiver, simply as the product by matrix $C^H$. Therefore, the despreader (or correlator) output is

$$Y_j = \frac{2}{N} X_j C^H = d(j) A S(\tau) + E_j$$

where the “noise” term is $E_j = 2/N V_j C^H$ and we have used that $C C^H \approx N/2 I$, which is a very good approximation for the long codes used in GNSS. It is straightforward that the columns of $V_j$ are temporally white too, and their spatial correlation matrix is $Q \triangleq 2/N Q$. The resemblance between the signal models in (6) and (8) unifies the description of the estimators using the signals before or after the despreading. All the following developments assume the model in (8) is employed, but their extension to the use of the signals before the despreading is immediate. In general, processing the despreaded signals may have some advantages, specially when the system is under-modeled (i.e. the assumed value for $d$ is smaller than the true one). In the derivations below, the signals during one symbol interval are considered. Therefore, the subscript $j$ is dropped, knowledge of $d(j)$ is not needed, and the symbol is included in the matrix $A$. All the results presented herein can be extended to the use of several symbol intervals, whether the symbols are known or not. In the first case, a new matrix $Y$ formed by the concatenation of $Y_j, Y_{j+1}...$ is built. In the second case, an ML estimator can be derived by considering that each matrix $Y_j$ is associated with a different $A_j$, but with the same $Q$.

3 Maximum Likelihood Estimators

In this section we present the ML estimators of the delays $\tau$ and amplitudes $A$. Although only the delay and amplitude of the LOSS may be of interest in GNSS receiver, in order to prevent the reflections from biasing the estimates, their parameters also have to be computed. The negative log-likelihood function of the data in (8) is

$$\Lambda_1(\tau, A, Q) = \ln |Q| + \frac{1}{L} \text{Tr} \left\{ Q^{-1} (Y - A S(\tau)) (Y - A S(\tau))^H \right\}$$

where $|\cdot|$ and $\text{Tr} \{ \cdot \}$ denote the determinant and the trace, respectively.

3.1 Spatially White Noise

If it is assumed that the noise received by the antenna array has not a spatial structure or directional properties, the noise correlation matrix is $Q = \sigma^2 I$ for some scalar $\sigma^2$. It is well known in this case the parameters that maximize (9) are [4]

$$\hat{A} = \hat{R}_{ww}(\tau) \hat{R}_{ss}^{-1}(\tau) \quad \hat{\tau}_{ML} = \arg \max_{\tau} \Lambda_w(\tau) \triangleq \arg \max_{\tau} \text{Tr} \left\{ Y P_{S(\tau)} Y^H \right\}$$

where

$$\hat{R}_{ww}(\tau) = \frac{1}{L} Y S^H(\tau), \quad \hat{R}_{ss}(\tau) = \frac{1}{L} S(\tau) S^H(\tau), \quad P_{S(\tau)} Y^H = S^H(\tau) (S(\tau) S^H(\tau))^{-1} S(\tau)$$

The second term in (10) involves the maximization of a function that only depends on the delays. Once the delays are estimated, the matrix $\hat{A}$ can be found in closed form. The estimators in (10)
are the extension of the minimum estimating delay loop (MEDLL) [2] when an antenna array is used. However, in the MEDLL a cost function that depends on both the delays and amplitudes is used. Then, the maximization is performed iteratively (some parameters are fixed while maximizing with respect to the others), which may result in converge problems. The interesting property of the cost function in (10) is that given an estimate of the delays, new estimates can be computed in closed form (i.e. without multidimensional searches). This is possible because if a frequency representation of the signals is used, then the matrix \( P_{\theta_{ss}}(\tau) \) can be reparameterized according to the coefficients of a certain polynomial and the cost function can be maximized using the iterative quadratic ML algorithm (IQML) [4], which is computationally efficient.

3.2 Spatially Correlated Noise

In order to obtain a estimator of the delays that is robust against directional interferences, we assume that the matrix \( Q \) is unknown and arbitrary. The value of \( Q \) that maximizes (9) is

\[
\hat{Q} = \frac{1}{L} \left( Y - A S(\tau) \right) \left( Y - A S(\tau) \right)^H
\]

which can be substituted back in \( \Lambda_1 \) and we get [3]

\[
\Lambda_2 (\tau, A) = \ln \left| \hat{R}_{yy} + A \hat{R}_{ss} (\tau) A^H - A \hat{R}_{ys} (\tau) - \hat{R}_{ys} (\tau) A^H \right|
\]

where \( \hat{R}_{yy} = \frac{1}{L} Y Y^H \). It is not difficult to show [3] that the estimator of the matrix \( A \) coincides with the one for the case of white noise, given in (10). If that value is substituted in (12) and (13), it results that the estimate of the correlation matrix is

\[
\hat{Q} = \hat{R}_{yy} - \hat{R}_{ys} (\tau) \hat{R}_{ss}^{-1} (\tau) \hat{R}_{ys} (\tau)
\]

and finally the delays estimate is given by

\[
\hat{\tau}_{ML} = \arg \min_{\tau} \Lambda_c (\tau) \triangleq \arg \min_{\tau} \left| I - \frac{1}{L} \hat{R}_{yy}^{-1/2} Y P_{\theta_{ss}}(\tau) Y^H \hat{R}_{yy}^{-1/2} \right|
\]

where the determinant identity \( |I + MN| = |I + NM| \) has been used in obtaining (15) from (13).

3.3 An Asymptotically Equivalent Estimator

In the estimation of the delays using (15) the received signals are filtered spatially in order to reduce the effect of the interference. The drawback is that the complicate dependence of \( \Lambda_c (\tau) \) on the delays, specially due to the presence of the determinant, implies that multidimensional searches or gradient-type algorithms are the only methods that may be used to find the estimates. Since these are not computationally efficient methods, we would like to transform the original criterion \( \Lambda_c (\tau) \) into another one that admits the use of IQML. To this end, let assume that an consistent estimate of the delays, denoted \( \hat{\tau} \), is available. Using (14) we can compute an estimate, \( \hat{Q} \), of the spatial correlation matrix. Then, it can be proved [6] that the estimates obtained as

\[
\hat{\tau} = \arg \max_{\tau} \Lambda_c (\tau) \triangleq \arg \max_{\tau} \text{Tr} \left\{ \hat{Q}^{-1/2} Y P_{\theta_{ss}}(\tau) Y^H \hat{Q}^{-1/2} \right\}
\]

asymptotically have the same variance as the exact ML estimates provided by (15). Though asymptotically refers to the number of samples, \( L \) or \( N \), tending to infinity, simulation results have shown that the performance achieved with \( \Lambda_c (\tau) \) and \( \Lambda_c (\tau) \) is also the same for a small number of samples, and no improvement is achieved by iterating the estimator (16). The advantage in using (16) is that the maximization can be done by means of the IQML algorithm because the dependence with \( P_{\theta_{ss}}(\tau) \) is linear. This fact is useful as long as the initial \( \tilde{\tau} \) may also be computed efficiently. Indeed this is possible, and it can be shown [6] that the estimator

\[
\tilde{\tau} = \arg \max_{\tau} \Psi (\tau) \triangleq \arg \max_{\tau} \text{Tr} \left\{ \hat{R}_{yy}^{-1/2} Y P_{\theta_{ss}}(\tau) Y^H \hat{R}_{yy}^{-1/2} \right\}
\]

satisfies that: i) it is consistent, ii) it allows the application of an IQML-like algorithm, iii) though not being efficient, it has interference cancelation capability.
3.1 Cramér-Rao Bound

An explicit expression for the Cramér-Rao Bound (CRB) of the time-delays is of interest because it is asymptotically achieved by (15) and (16). Using the Slepian-Bangs' formula [7] and some algebraic effort, the following expression can be derived

\[ \text{CRB}^{-1}(\tau) = 2 \text{Re} \left\{ \left( H \mathbf{P}^\dagger S\mathbf{H} \right) \odot (\mathbf{A}^H \mathbf{Q}^{-1} \mathbf{A})^T \right\} \]  

(18)

where \( \odot \) represents the Hadamard (element-wise) product and

\[ H(\tau) = \begin{bmatrix} h(\tau_1) & \cdots & h(\tau_d) \end{bmatrix}^T \quad h(\tau_i) = \frac{\partial}{\partial \tau} s(\tau) \quad \mathbf{P}^\dagger S\mathbf{H}(\tau) = \mathbf{I} - \mathbf{P} S\mathbf{H}(\tau) \]  

(19)

The same expression is valid for the white noise case if \( \mathbf{Q} \) is replaced by \( \sigma^2 \mathbf{I} \). On the other hand, a simple expression for the CRB of \( \mathbf{A} \) cannot be calculated.

4 Simulation Results

In this section we present some numerical results for a system with the following parameters. The chip-shaping pulse is a square-root raised-cosine pulse with 0.2 roll-off. This is the pulse proposed for the GNSS2 [1]. Its length is truncated to \( L = 13 \) samples. The length of the PN code is \( N = 20 \cdot 1023 \) chips and CNo = 44dB Hz. We use an uniform linear array with antennas spaced 0.5\( \lambda \) apart. The direct signal and a reflection are present at DOAs 0° and 10°. The reflection is attenuated 3dB with respect to the direct signal, and both signals are in phase at the first sensor. Fig 1 and Fig 2 display the bias and the variance of the white-noise estimator (10), \( \Lambda_w \), and the correlated-noise estimator (15), \( \Lambda_c \), when they assume a model with \( d = 1 \). If the correct model order (\( d = 2 \)) is chosen, none of the estimators is biased or, at least, the bias is much smaller than the variance, shown also in Fig 2. Note that \( m = 4 \) antennas are used in Fig 2. It is clear from Fig 1 that the bias decreases with the number of antennas. The bias resulting from \( \Lambda_c \) is much smaller than that from \( \Lambda_w \). If these cost functions were plotted, we could observe that the latter fails in resolving the two signals (only has one peak), meanwhile the former resolves the signals most of the times. The price to pay is an increased variance of the estimates provided by \( \Lambda_c \). However, for \( d = 1 \) the overall performance (i.e., considering the bias and variance) of \( \Lambda_c \) is much better than that of \( \Lambda_w \). For a model order \( d = 2 \), both criteria exhibit the same variance, which means that it is no degradation due to the estimation of the noise correlation matrix. Their variance tends to infinity as the delay of the reflection is reduced. This implies that only when the delays of the direct signal and the reflection are extremely close (e.g., \( \tau_1 - \tau_0 < 0.02 \tau_c \)) the

![Figure 1: Bias for an underparametrized model.](image1)

![Figure 2: RMSE for well- and under- parameterized models. \( m = 4 \) antennas.](image2)
use of the estimator derived under the model \( d = 1 \) is preferred. In Fig 3 and Fig 4, \( m = 6 \) antennas, the reception of a wideband interference from DOA \(-30^\circ\) is considered, and the time-delays are \( 0T_c \) and \( 0.4T_c \). Fig 3 shows the root mean squared error (RMSE) of different estimators as a function of the number of training symbols when the signal-to-interference ratio (SIR) after the despreading is 3dB. In Fig 4 the RMSE is plotted against the SIR. As expected, the white-noise criterion \( \Lambda_w \) suffers a severe degradation in the presence of interferences. Indeed, it fails completely for SIR < \(-5\)dB. An important result is that \( \Lambda_c \) and its efficient approximation \( \Lambda_e \) in (16) have the same RMSE. Finally, the consistent estimator \( \Psi \) in (17) has a slightly larger RMSE than the two previous estimators, but the key fact is that it is also robust against arbitrarily strong interferences.

5 Concluding Remarks

Two time-delay estimators have been presented and compared. Both of them assume that several delayed reflections are received. The spatial signatures of all the signals are taken as unstructured and unknown. The first estimator models the received noise as spatially white, meanwhile the other assumes that the noise correlation is unknown. This second approach always outperforms the first one, specially when the system is undermodeled and/or external interferences are received. Moreover, we have proposed an approximation of the estimator for the correlated-noise case that allows the use of computationally simpler algorithms with no performance degradation.

References


