ITERATIVE ALGORITHM FOR THE ESTIMATION OF DISTRIBUTED SOURCES LOCALIZATION PARAMETERS

Antonio Pascual Iserte¹, Ana I. Pérez-Neira¹ and Miguel Ángel Lagunas Hernández¹,²

¹Department of Signal Theory and Communications, Polytechnic University of Catalonia (UPC)
²CTTC-Centre Tecnològic de Telecomunicacions de Catalunya
Edifici NEXUS I
C/ Jordi Girona 1-3 (Campus Nord UPC - mòdul D5), 08034 Barcelona (SPAIN)
e-mail: {tomip,anuska.miguel}@gups.tsc.upc.es

ABSTRACT

We present a novel algorithm for the estimation of the direction of arrival and angular distribution parameters of sources that, as a result from the scattering effects, cannot be considered as punctual.

The algorithm is iterative and is based on the maximization of the likelihood function associated to the received snapshots at the antenna array (ML). It is proposed a computationally efficient method for estimating the localization and angular distribution parameters of more than one source transmitting at the same frequency in a noisy environment. This algorithm solves the problem of the joint maximization in the case of two sources by formulating two new problems of single-source ML.

Key Words- Array signal processing, DOA estimation, distributed sources, statistical parameter estimation.

1. INTRODUCTION

Classically, the methods for the estimation of the direction of arrival (DOA) have considered punctual sources and spatio-temporal thermal white noise. This problem can be assumed equivalent to those of frequency detection based on temporal diversity. However, whereas in spectral analysis, two different frequencies are always totally uncorrelated, in spatial diversity two signals impinging from different angles can be partially correlated.

The multipath propagation implies an increase in the temporal correlation between signals from different directions, making the performance of the classical spectral analysis techniques worse. Besides the spatial smoothing techniques, the most effective solution to this problem is represented by the spreading systems, such as radar “pulse compression techniques” and spread spectrum communications (DSSS), which are based on an increase of the signal bandwidth. By means of these techniques, the classical non-parametric spectral analysis algorithms can be applied, converting the DOA detection in a multipath environment in multiple uncorrelated punctual sources DOA detection problems, even with minimum time shift differences between echoes.

However, the presence of scatterers near the transmitter with no relative delays between different DOAs, makes the performance of the spreading systems worse. The source must be considered as distributed, and therefore the classical methods may fail because of the high spatio-temporal correlation. As in the case of specular multipath, this problem cannot be solved by manipulating the transmitted signal. The distributed source signs with a unique temporal waveform and a spatial signature. Although the scattering may change over the time, it can be considered time-invariant within the frame duration in most of the cases. That means that in large periods of time, the correlation matrix of the received snapshots, considering a free-noise environment, is full-rank [1] [2], whereas in short periods, due to the local analysis of the scenario, each source is contributing as a rank one covariance matrix, so it is completely coherent during a frame. Our interest is to characterize this quasi-static behaviour of a source, and not the inter-frame or inter-scan changes [1] [2] [3].

The proposed technique consists in maximizing the likelihood function associated to the parameters of the angular distribution of the sources. This represents a multi-variable maximization problem with a very high computational cost. Solutions based on EM (Estimate & Maximize) [4] [5] or AP (Alternating Projection) [6] and RCAP (Reduced Complexity Array Processing) [7] [8] convert this problem into multiple one-dimensional problems.

This paper presents an algorithm for the estimation of the spatial signature of multiple distributed sources with rank one contributions, that is, estimation within the time duration of a frame. In the case of a more prolonged observation period, the classical spectral analysis methods can be applied. In general terms, this work presents the generalization of the AP and RCAP techniques to the case of distributed sources.

2. SIGNAL MODEL

In the case of a single source scenario and an array of antennas, a known angle distribution can be assumed $f_0(\theta, n)$, whose mean is $\theta_0$ and $n$ is the temporal index of the received snapshot. The snapshot model is as follows [9]:

\[ x_n = a(n) \int_{-\pi/2}^{\pi/2} f_0(\theta, n)s(\theta)d\theta + w_n = a(n)\mathbf{b}_n + \mathbf{w}_n \]

\[ \mathbf{b}_n = \int_{-\pi/2}^{\pi/2} f_0(\theta, n)s(\theta)d\theta \]  

(1)

where $a(n)$ is the complex envelope of the transmitted signal, $s(\theta)$ is the steering vector for a punctual source in the elevation angle $\theta$ and $\mathbf{w}_n$ is the noise contribution at the front-end. In this model the complex envelope is the same for all the angles of arrival of the source, so it is totally correlated. The goal is to estimate the spatial signature $\mathbf{b}_n$ of the source, which is already defined in (1).

In the case of long or inter-frame observation periods, the spatial signature can change, and so its temporal correlation can be
exploited by the system to update the estimate of the source movement or position, in the case of low-mobility environments. The inter-frame spatial signature tracking system can be based on a Kalman filter, although this is outside the scope of this work [8].

Our interest is centered in the spatial signature estimate for the case of non-varying sources. This is the general situation in a space-time diversity scheme at the receiver and/or the transmitter side of the communication system, in which an estimate of $b$ is required within a frame duration. Although this signature can change in time periods longer than the frame duration, it can be assumed constant within one frame and the $N$ received snapshots, so the signal model is as follows:

$$\mathbf{x}_n = \alpha(n) \mathbf{b} + \mathbf{w}_n \quad (2)$$

where now it is remarked that the spatial signature $\mathbf{b}$ does not depend on the temporal index $n$. This is the same situation as the one described in [2] for the case of Coherently Distributed Sources. In that case, the angular distribution $f_0(\theta)$, called deterministic angular signal density, is assumed to be known.

In order to estimate the spatial signature, the preamble and reference symbols in the frame may be used. In the problem we assume, no reference signal is used. In this case, and taking equation (2) as a basis, the estimated covariance matrix for a scenario with $N_S$ independent sources is as follows:

$$\hat{\mathbf{R}} = \frac{1}{N} \sum_{n=0}^{N-1} \mathbf{x}_n \mathbf{x}_n^H \approx \sum_{j=1}^{N_S} \alpha_j \mathbf{b}_j \mathbf{b}_j^H + \sigma^2 \mathbf{I} \quad (3)$$

$$\sigma^2 = E\left\{ \mathbf{w}_n \mathbf{w}_n^H \right\} \quad \alpha_j = E\left\{ |\alpha_j(n)|^2 \right\}$$

It will be assumed in the theoretical description of the algorithm, that there is no error in the estimation of the covariance matrix. In the simulations, the impact of the variation of the number of snapshots will be shown.

Our goal is the estimation of the parameters of the distribution $f_0(\theta)$. It is practical to consider a discrete model of the spatial signature instead of its integral model (1). We consider the contribution of the same signal impinging from $M$ different angles, where $M$ is much larger than the total number of sensors $Q$ [9]:

$$\mathbf{b} = \sqrt{Q} \frac{\mathbf{S} f_0}{\| \mathbf{S} f_0 \|} \quad \Rightarrow \quad \mathbf{b}^H \mathbf{b} = 1 \quad (4)$$

where $\mathbf{S} = \begin{bmatrix} s(\theta(0)) & s(\theta(1)) & \cdots & s(\theta(M-1)) \end{bmatrix}$ and we have normalized the spatial signature $\mathbf{b}$ so that the mean signal power measured at the sensors of the array is equal to the signal power $\alpha_\theta(\theta(k))$ represents the discretized angular axis.

At this point, it is only necessary to parametrize the distribution. The experimental results and measurements at 2 GHz indicate that the best-fitting model is the exponential one, with its mean value situated at the real position of the source. The discretization of the angular distribution is expressed in the vector $\mathbf{f}_0 = \begin{bmatrix} f_0(\theta(0)) & f_0(\theta(1)) & \cdots & f_0(M-1) \end{bmatrix}^T$, where $f_0(m) = \exp(-\alpha_\theta(\theta(m) - \theta_0))$ $0 \leq m \leq M - 1$. In the simulations, not only a Laplacian, but also Gaussian and rectangular profiles have been proved. In all the cases, the performance of the algorithms described throughout the paper is good. In the simulations, the results for the case of a Laplacian distribution are shown.

In section 3 our goal is to estimate the spread parameter $\alpha_\varnothing$ and the mean angle of arrival $\theta_0$ based on the estimate of the covariance matrix in the case of a single-source scenario. In the next sections, the generalization to the case of a two sources environment is discussed through algorithms based on AP and RCAP.

### 3. SINGLE SOURCE ESTIMATION

It is well known that the maximum likelihood (ML) estimator of the spatial signature $\mathbf{b}$ in an AWGN environment is as follows:

$$\hat{\mathbf{b}} = \arg \max \frac{1}{2} \mathbf{b}^H \hat{\mathbf{R}} \mathbf{b} \quad \Rightarrow \quad \hat{\mathbf{b}} = k \mathbf{e}, \quad \hat{\mathbf{R}} \mathbf{e} = \lambda_{\max} \mathbf{e}, \quad \| \mathbf{e} \| = 1 \quad (5)$$

so, it is an eigenvector problem, where the maximum eigenvalue must be chosen. Due to errors in the estimation of the covariance matrix, it is possible that the eigenvector $\mathbf{e}$ does not exactly fit the parametric definition of the spatial signature $\mathbf{b}$ as expressed in (4). A MSE (Minimum Square Error) criterion is proposed so as to fit the distribution parameters to the eigenvector $\mathbf{e}$:

$$\hat{\theta}_0, \hat{\alpha}_\varnothing, \hat{\beta}_0 = \arg \min \left( \sqrt{\lambda_{\max} \mathbf{e} - \beta_0} \mathbf{b}(\hat{\theta}_0, \hat{\alpha}_\varnothing) \right)^2 \quad (6)$$

where $|\beta_0|$ is an estimate of the RMS value of the source and $\mathbf{b}$ is defined as shown in (4). The validity of the expression is based on the idea that the maximum eigenvalue is an approximated measurement of the source power in a single source and typical SNR environment. In the case of extremely low SNR conditions, a noise calibration should be carried out.

The mean value of the angular distribution can be easily estimated through the following expression, where the spatial response of the eigenvector $\mathbf{e}$ is calculated:

$$\hat{\theta}_0 = \arg \max \left( s^H(\hat{\theta}_0) \mathbf{e} \right)^2 \quad (7)$$

We admit that this estimator has a bias. However, for sources situated within the angle view $[\theta(0), \theta(M-1)]$ and typical spreadings in mobile communications, the deviation is minimum. The great advantage is that, making use of this estimator, a two dimensional search (mean angle and spreading parameter) is avoided in (5).

The estimate of the parameter $|\beta_0|$ can be expressed in function of the estimate of the spatial signature, where now, only an unidimensional search on the spreading parameter $\alpha_\varnothing$ is necessary based on the MSE expression (6).

$$\tilde{\mathbf{b}} = \sqrt{Q} \frac{\mathbf{S} f_0}{\| \mathbf{S} f_0 \|} \quad \tilde{f}_0 = f_0(\hat{\theta}_0, \hat{\alpha}_\varnothing) \quad \tilde{\beta}_0 = \sqrt{\lambda_{\max}} \tilde{b}^H \tilde{\mathbf{e}} \quad (8)$$

At this point, it is important to highlight that the traditional methods suffer important degradations when trying to estimate parameters of distributed sources using the classical punctual source model. It is the same degradation as that produced by a system with an uncalibrated array.

### 4. TWO SOURCES ESTIMATION

Now the previous method is extended to the case of a scenario with two distributed sources. The extrapolation of the algorithm to the case of more sources is direct, although the computational cost grows importantly. In most of the real communication systems, it is not exaggerated to assume that only two independent sources with no negligible power level are radiating, where one of them can be considered as the desired signal and the other as the interference.

In the following presentation, the spatial signatures $\mathbf{b}_1$ and $\mathbf{b}_2$ will be used. The algorithm is iterative, and in each step the
parameters of one of the sources are estimated, based on the previous estimates of the other source parameters. It can be shown in the simulations that the algorithm converges and the quality of the estimates improves as the number of iterations grows. In this paper, one of the steps is presented, where a previous estimate of the second source is assumed:

\[
\tilde{b}_2 = \frac{1}{\sqrt{q}} \frac{S_{\tilde{f}_2}}{S_{f_2}} \tilde{f}_2 = f_0(\tilde{\theta}_2, \tilde{\varphi}_2) \quad \tilde{\beta}_2 = \sqrt{\lambda_{\max}} \frac{\tilde{b}_2^H e}{b_2^H b_2} \quad (9)
\]

Based on this estimate, it is possible to estimate the covariance matrix \( \tilde{R}_1 \) of the snapshots without the contribution of the second source. The algorithm presented in section 3 is now applied to \( \tilde{R}_1 \) so as to estimate the parameters of the first source. The mechanism must be applied iteratively to obtain admissible estimates.

\[
\tilde{R}_1 = \tilde{R} - \tilde{\beta}_2^2 \tilde{b}_2 \tilde{b}_2^H \quad (10)
\]

It is interesting to comment a conflicting point in this algorithm, which deals with the second source RMS power estimate \( \tilde{\beta}_2^2 \) (9), similar to the one deduced by Li and Stoica [10]. Equation (10) does not guarantee that \( \tilde{R}_1 \) is positive defined. The maximum source power estimate that guarantees it is \( (b_2^H \tilde{R}^{-1} \tilde{b}_2)^{-1} \) while the MSE based \( \tilde{\beta}_2^2 \) estimate (6) (9) may be greater. However, in the simulations it will be proved that the algorithm works well, although some of the eigenvalues may be negative in the first steps of the iterative mechanism.

The algorithm presented up to this point and based in (10) is called “subtraction method”. Now, we present another method based on a blocking matrix as the AP [6] or RCAP [8] algorithms, whose name is “blocking method”. In this case we consider the second source as Gaussian noise with a known covariance matrix. Our goal is to estimate the parameters of the first source based on the maximization of the log-likelihood associated to the received snapshots. Taking into account all the snapshots, and assuming that the symbols emitted from the second source are independent and the noise is white, the estimated spatial signature for the first source is calculated as follows (see Appendix):

\[
\tilde{b}_1 = \arg \max \frac{\tilde{b}_1^H P \tilde{R} P \tilde{b}_1}{b_1^H P b_1} \Rightarrow \tilde{b}_1 = k e, \quad (11)
\]

\[
\tilde{R} \tilde{P} e = \lambda_{\max} e, \quad \|e\| = 1
\]

where \( P \), called blocking matrix, is defined as follows:

\[
P = I - \phi_2 \tilde{b}_2 \tilde{b}_2^H \quad (12)
\]

\[
\phi_2 = \frac{SNR_2}{1 + b_2^H b_2 SNR_2}, \quad SNR_2 = \frac{|\tilde{\beta}_2|^2}{\sigma^2}
\]

Equation (11) is a modified eigenvector problem very similar to the one described in the case of AP or RCAP algorithms. As explained in section 3, when the eigenvector \( e \) is calculated, the spatial signature \( b_1 \) and the distribution parameters of the first source (mean DOA and spreading parameter) must be fitted as expressed in the next equation, based on a MSE criterion:

\[
[\tilde{\theta}_1, \tilde{\varphi}_1, \tilde{\beta}_1] = \arg \min \left\| \frac{\lambda_{\max}}{1 - \phi_2} e^{H} b_2 - \tilde{\beta}_1 b_1(\tilde{\theta}_1, \tilde{\varphi}_1) \right\|^2 \quad (13)
\]

The constant multiplying the eigenvector is found by calculating the maximum eigenvalue of the matrix \( \tilde{R} P \) which is the one that solves equation (11): \( \lambda_{\max} = |\beta_1|^2 |b_1|^2 - |\beta_2|^2 \phi_2 |b_2|^2 \), assuming a sufficiently high SNR environment.

The blocking matrix is defined by the parameter \( \phi_2 \), which depends on the level of the source to be blocked. In the case of AP or RCAP, this blocking is independent of the source level. In this sense, the performance of the algorithm presented in this paper copes better with variations of the source levels. Only for high SNR conditions, the blocking matrix is equal to the case of AP and RCAP:

\[
SNR_2 \to \infty \quad \phi_2 \to \frac{\| \tilde{b}_2 \|^2}{2} \quad (14)
\]

If more than two sources are considered, the matrix \( P \) is defined as the inverse of the correlation matrix of the noise plus the \( N S - 1 \) source signals different from the one whose parameters are being estimated. This can be easily deduced from the Appendix.

5. Simulations and Results

Some simulations and results about the performance of the algorithms are now presented. We consider the presence of two independent distributed sources at 8º and -10º, with spreading parameters 0.5 and 0.1 respectively. The smaller the spreading parameter is, the more distributed the source is.

The next figure shows the RMS error in the mean angle estimation of the two sources as a function of the number of antennas and the algorithm. The SNR for each source is 10 dB, and the number of snapshots used to estimate the covariance matrix is 100. 100 simulations have been carried out per point in the curves.

![RMS error in angle estimation](image)

Fig. 1. RMS error vs. number of antennas.

It can be observed that the angle estimation of the source at -10º, which is the most distributed, presents higher error. It can be also seen that the blocking method performs better than the subtraction method.

The impact of the SNR variation on the estimation performance of the spreading parameters is shown in Fig. 2. The simulation parameters are the same as in the previous figure, where now 8 antennas are used and both sources have the same level. SNR refers to the signal-to-noise ratio of each source. For high SNR...
conditions, both methods perform similar.

\[ R_2 = \sigma^2 I + [\beta_x]_2^T \beta_2 b_2^T P - K_2 R_2^{-1} - \phi_2 b_2 b_2^T \]

The log-likelihood is expressed as follows \( \Lambda_x = \log (f(x)) \):

\[ \Lambda_x = K - \sum_{n=0}^{N-1} (x_n - a_1(n)b_1)^H R_2^{-1} (x_n - a_1(n)b_1) \]

Maximizing with respect to the information symbols, they can be estimated as expressed in the next equations:

\[ \hat{a}_1(n) = \frac{b_1^H R_2^{-1} x_n}{b_1^H R_2^{-1} b_1} \]

\[ P_1 = \frac{1 - b_1 b_1^H P}{b_1^H P b_1} \]

\[ \Lambda_x = K - \sum_{n=0}^{N-1} x_n^H P_1^H R_2^{-1} P_1 x_n = K - \frac{N}{K_2} tr \left( P_1^H P R_1 \right) \]

\[ = K - \frac{N}{K_2} \left\{ tr \left( R P \right) - tr \left( \frac{b_1 b_1^H P R P b_1}{b_1^H P b_1} \right) \right\} \]

The estimate of the spatial signature for the first source is obtained by maximizing \( \Lambda_x \):

\[ b_1 = \arg \max \frac{b_1^H P R P b_1}{b_1^H P b_1} \Rightarrow R P e = \lambda_{\max} e, \quad ||e|| = 1 \]

7. REFERENCES


