A Joint Transmitter-Receiver Design in MIMO systems Robust to Channel Uncertainty for W-LAN Applications

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ABSTRACT

The performance of closed-loop schemes, assuming perfect knowledge of the channel response at the transmitter and receiver, degrades when available Channel State Information (CSI) is noisy. In this paper a MMSE criterion algorithm taking into account the model for the error in the CSI is developed and a closed form solution is found for it. The proposed algorithm has been designed for MIMO systems with OFDM modulation for Wireless-LAN applications.

I. INTRODUCTION

Closed-loop multiple antennas systems are growing in importance because its high capacity allows to achieve high data rate transmission. These schemes are well suited to Wireless-LAN applications thanks to the concurrence of low mobility scenarios and TDD transmission (i.e. high coherence time and reciprocity), high information rate (i.e. need of high capacity links) and OFDM modulations (i.e. simple procedures for optimal power allocation). Nevertheless, to fully exploit the potential of these MIMO (multiple-input multiple-output) schemes, Channel State Information (CSI) should be available at both transmitter and receiver. Several algorithms have been proposed assuming that the channel is perfectly known at both ends of the transmission link [1]-[3]. However, in real environments, CSI is never completely available, either because there is some error in the estimation process or because the system is working with outdated values. Consequently, although closed-loop MIMO systems can theoretically achieve high performances, in practical scenarios, the behaviour of these algorithms will degrade if ideal CSI is assumed. The sensitivity of existing algorithms to errors in CSI as well as the design of robust solutions is still an open question. One of the few references assuming non-ideal CSI is [1], where the authors deal with the single antenna OFDM case, and propose a model for the CSI based on the channel estimation as a linear predictor. Their design aims to optimize the SNR at the receiver, regarding residual ISI due to channel uncertainty as noise.

This paper proposes a new technique for optimal power allocation in scenarios where channel estimates are noisy. The optimization follows a MMSE criterion that includes the model of the errors in the CSI. Thus, an algorithm that is robust in front of channel estimation errors is derived. As will be shown, its formulation and its complexity is similar to the non-robust original algorithm, including one SVD computation per carrier. The algorithm is formulated for a general MIMO multicarrier system, and includes as particular cases the frequency-flat-fading (FF) MIMO scenario and the single antenna OFDM scenario. The application of these techniques to MIMO in the field of DMT (Discrete Multi-Tone) modulation was addressed in [4], but the authors assumed ideal CSI.

The paper is organized as follows. Next section describes the system model of the MIMO-multicarrier system. Section III presents the cost function of the proposed algorithm, and derives a closed form solution. Finally section IV shows the performance of the new robust algorithm and compares it with existing solutions.

II. SYSTEM MODEL

In this section we summarize the signal model for a MIMO communications system with $M_T$ transmitter and $M_R$ receiver antennas using OFDM modulation with $N$ carriers. The design is developed in the frequency domain allowing us to decouple the frequency-selective MIMO channel into $N$ MIMO frequency-flat channels by imposing a certain structure in the transmitter and receiver matrices.

Figure 1 illustrates a block diagram of the proposed transmission scheme. Let $\mathbf{x}^T = [\mathbf{x}(1)^T \ldots \mathbf{x}(M)^T]$ be the $M \cdot N \times 1$ vector that simultaneously stacks $M$ blocks $\mathbf{x}(i)$, where $M = \min \{M_T, M_R\}$, and $\mathbf{x}(i)$ contains $N$ information symbols to be transmitted. The information symbols

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are assumed to be i.i.d. with zero mean and unit variance. Assuming that the channel keeps invariant within one OFDM symbol and the cyclic prefix (whose length is appropriately chosen) is removed at the receiver, the system model can be written in the frequency domain as:

$$y = GHFx + Gn$$  \hspace{1cm} (1)

where \(H\) is the block diagonal channel matrix containing the frequency responses of the MIMO channels; \(F\) becomes a block matrix, arranged by diagonal submatrices, which sets the power allocation over the \(N\) carriers and \(M_T\) antennas; \(G\) has the same structure as \(F\), and combines the signal received at the \(M_R\) antennas; and \(n\) is the noise vector after the FFT, which has the same statistics as its time-domain counterpart \(E\{nn^H\} = \sigma_n^2I\).

Premultiplying and postmultiplying the channel matrix by the appropriate permutation matrices, (1) can be reordered to express the input-output relation in terms of block diagonal matrices that involve only one subcarrier each. Hence \(G,H,F\) and \(x,y\) can be structured as:

$$H = \text{diag}\{[H_1,H_2\ldots H_N]\}$$
$$F = \text{diag}\{[F_1,F_2\ldots F_N]\}$$
$$G = \text{diag}\{[G_1,G_2\ldots G_N]\}$$
$$x^T = [x_1^T\ldots x_N^T]$$
$$y^T = [y_1^T\ldots y_N^T]$$  \hspace{1cm} (2)

where \(F_1, H_2, \text{and } G_3\) are respectively \(M_T \times M\), \(M_R \times M_T\) and \(M \times M_R\) matrices processing the \(k\)-th carrier, and \(x_k, y_k\) are \(M \times 1\) vectors containing the symbols transmitted/received by the subcarrier \(k\) through the different antennas. Notice that the reordered system decouples all subcarriers, obtaining \(N\) independent equations:

$$y_k = G_kH_kF_kx_k + G_kn_k \quad k = 1 \ldots N$$  \hspace{1cm} (3)

Equations (2)-(3) summarize the notation that will be used for the development of the algorithm in the frequency domain.

### III. ALGORITHM DESIGN

#### A. Cost function

The proposed algorithm designs matrices \(F_k, G_k\) according to a MMSE criterion taking into account the channel uncertainty. The algorithm models the error in the CSI as a Gaussian variable with known statistics, uncorrelated from the true channel response. This knowledge is incorporated into the optimization cost function to mitigate the impact of this error on algorithm performance. In this section the derivation of the cost function and its optimization is outlined, a detailed description can be found in [5].

The channel response estimate \(\hat{H}_k\) can be expressed in terms of the true value as

$$\hat{H}_k = H_k + \varepsilon_k$$  \hspace{1cm} (4)

where \(E\{\varepsilon_k\varepsilon_k^H\} = \sigma^2I\) because the antenna fading is assumed to be uncorrelated, and \(E\{\varepsilon_k\varepsilon_k^H\} \neq 0\) due to the correlation between subcarriers. This correlation can be used to reduce the uncertainty in the design of \(F_k, G_k\) as will be shown in Appendix I.

The proposed algorithm follows a MMSE criterion subject to an average transmitted power constraint:

$$\min_{F_k,G_k} E_{\varepsilon_k} \left\{ (y - x_k)^H \hat{H} \right\} =$$

$$= \sum_{k=1}^{N} E_{\varepsilon_k} \left\{ E_{\varepsilon_k \varepsilon_k^H} \left\{ (y_k - x_k)^H x_k \hat{H} \right\} \right\}$$

$$= \sum_{k=1}^{N} E_{\varepsilon_k} \left\{ E_{\varepsilon_k \varepsilon_k^H} \left\{ y_k^H x_k - x_k^H x_k \right\} \hat{H} \right\}$$  \hspace{1cm} (5)
\[ \sum_{k=0}^{N-1} \text{tr} \left\{ F_k F_k^H \right\} = P_0 \]  

(6)

Introducing the statistics of \( y \) conditioned to \( x_k \) and \( \hat{H} \) (see Appendix I) the mean squared error for the \( k \)-th carrier can be expressed as:

\[ \zeta_k^2 = E_{x_k} \left\{ \left( \hat{y}_{k} - x_k \right)^2 \right\} = \bar{w}_{k\|} \cdot \text{tr} \left\{ F_k F_k^H \right\} \cdot \text{tr} \left\{ G_k G_k^H \right\} + \sigma_n^2 \right] + \text{tr} \left\{ \left( \hat{G}_k \hat{H}_k^\ast F_k - I \right)^{\ast} \right\} \]  

(7)

\[ \bar{w}_{k\|} \cdot \text{tr} \left\{ F_k F_k^H \right\} \cdot \text{tr} \left\{ G_k G_k^H \right\} + \sigma_n^2 \]  

(8)

where \( \bar{w}_{k\|} \) is a measure of the channel error and \( \hat{H}_k^\ast \) an equivalent channel, which tries to mitigate the channel uncertainties making use of the correlation between carriers (see Appendix I for definition of \( \bar{w}_{k\|} \) and \( \hat{H}_k^\ast \)).

The optimization criterion can be expressed as:

\[ \min_{F_k, G_k} \sum_{k=0}^{N-1} \zeta_k^2 \text{ subject to } \sum_{k=0}^{N-1} \text{tr} \left\{ F_k F_k^H \right\} = P_0 \]  

(9)

that can be solved by means of Lagrangian multipliers.

It is interesting to analyze the similarities between equations (8)-(9) and the cost function appearing in the non-robust solution (e.g., [2] eqn. (7)-(9)). On the one hand, the noise power in the second term in (8) and the residual ISI in the third term also appeared in [2], but now the true channel has been replaced by its estimate \( \hat{H}_k^\ast \). On the other hand, the first term will only be zero if there is no uncertainty, i.e. \( \sigma_n^2 = 0 \).

B. Closed form solution

In this paper, two different procedures are proposed to optimize the previous cost function (9), an iterative solution and a closed form solution. The iterative procedure was already introduced in [6] and has the advantage that it avoids the SVD computation and it always provides non negative solutions for \( \Phi_k \Phi_k^H \). However, for the problem under analysis, a closed form solution has been found as it is described below.

It can be shown that the minimization of the cost function can be achieved selecting matrices \( F_k \) and \( G_k \) to have the following structure according to the SVD of the equivalent channel matrix \( \hat{H}_k^\ast \) for each subcarrier:

\[ \hat{H}_k^\ast = U_k \Sigma_k V_k^H \]  

\[ F_k = V_k \Phi_k \]  

(10)

\[ G_k = \Gamma_k \Sigma_k^\ast U_k^H \]  

where \( \Phi_k \), \( \Gamma_k \) are diagonal matrices, and \( ^\ast \) stands for the pseudoinverse. Replacing (10) in (8) and after some manipulations over the derivative with respect to \( \Phi_k \), \( \Gamma_k \) the following equation is obtained (see Appendix II):

\[ \left( \hat{W}_k \cdot \text{tr} \left\{ \Phi_k \Phi_k^H \right\} + \sigma_n^2 \right) I + \Sigma_k^2 \Phi_k \Phi_k^H = C \Sigma_k \]  

(11)

For each carrier, previous equality provides a set of \( M \) equations that are linear on the \( M+1 \) unknowns \( \Phi_k, C \). If these \( N \) sets of equations are combined with the power constraint they can be solved to find all the unknowns. Otherwise, their resolution can be simplified by solving the problem for each carrier in two stages as shown in Appendix II.

IV. SIMULATION RESULTS

Computer simulations were carried out to illustrate the robustness of the proposed closed-loop algorithm in the presence of channel estimation uncertainties. A \( M_T = 3 \), \( M_R = 3 \) MIMO scheme over a frequency selective fading channel was considered. According to the HIPERLAN/2 standard [7], the bit stream to be transmitted was coded with a convolutional encoder of coding rate 1/2, and generator polynomials 133oct and 171oct. The encoder was initialized to zero state and returned to it after encoding 3456 bits (i.e. 8 DLC-PDUs of 54 bytes - according to HL/2) by appending 6 tail bits. After the encoding process, the encoded bits were block interleaved as [7], mapped into a QPSK constellation, multiplexed in multiple antennas and modulated in OFDM symbols (including pilot tones and empty carriers according to HL/2 [7]).

The simulated channel modeled a typical indoor scenario with 100ns of delay spread and exponential power delay profile (PDP). Such channel was implemented (according to HL/2 parameters) with a normalized discrete channel sampled at \( f_s = 20 \text{ MHz} \).

The robustness of the proposed algorithm, both the iterative solution ("MMSE Cond B") and the closed form ("MMSE Cond") was compared with other algorithms that assume ideal CSI and do not take into account the estimation uncertainties: the closed loop scheme under a MMSE criterion ("MMSE"), and the open-loop scheme (uniform power distribution at the transmitter and MMSE equalizer at the receiver) ("MMMSE Rx".). Channel uncertainties were assumed constant for all taps in the channel impulse response and independent of the exponential PDP, as would be the case in a linear channel estimator [8]. Two simulations were carried out for different variance channel uncertainties \( \sigma_n^2 \). First simulation considered a scenario where channel identification error was large, i.e. \( \sigma_n^2 = 10^{-2} \) for the first 12 taps of the channel response, while second simulation considered a lower channel identification error, i.e. \( \sigma_n^2 = 10^{-3} \) for the first 12 taps of the channel response. The performance has been analyzed in terms of MMSE and BER. To allow the comparison between both criteria, all simulations has been plotted in terms of Eb/N0 before encoding (i.e. taking into account the R=1/2 coding rate).

The Viterbi algorithm was used for decoding of the convolutional code. Assuming the receiver has noisy channel estimates with known statistics, the ML decoder is based on the maximization of the following metrics:

\[ D(y, x) = \frac{B}{M} \sum_{k=0}^{N-1} 2 \text{Re} \left\{ y_k^H Q_k^{-1} T_k x_k \right\} \]  

(12)

where \( B \) is the number of transmitted symbols, and \( Q_k \) and \( T_k \) are diagonal matrices defined as:

\[ Q_k = \left( \bar{w}_{k\|} \cdot \text{tr} \left\{ \Phi_k \Phi_k^H \right\} + \sigma_n^2 \right) \Gamma_k \Sigma_k^\ast \Gamma_k^H \]  

(13)

\[ T_k = \Gamma_k \Phi_k \]  

A detailed analysis of (12-13) shows that when the power allocation procedure (i.e. \( F_k \) design) assigns no power to
a certain subcarrier (i.e. \( \phi_i = 0 \)), the corresponding element in \( \mathbf{T}_k \) equals to zero and consequently there is no contribution into the metrics of such carrier (equivalent to an erased symbol in the decoding procedure). Moreover, in perfect CSI, \( \bar{w}_{kk} \) (a measure of the channel knowledge) equals to zero and consequently only the noise contributes in the covariance matrix \( \mathbf{Q}_k \). Nevertheless, in the presence of channel uncertainties, \( \bar{w}_{kk} \) increases, resulting in high covariance terms and hence reducing the contribution into the metrics.

Figures 2 and 3 show the performance loss in terms of MSE due to mismatch between the real channel and the noisy CSI used to allocate the transmitted power. They show that the MSE of the non-robust algorithms (i.e. "MMSE" and "MMSE Rx") degrades as the Eb/No increases. On the contrary, the two robust solutions proposed in this paper exhibit an error floor.

Figure 4 plots the BER before the convolutional decoder ("Raw-BER"), and the BER after the Viterbi decoder ("Coded-BER"). As the aim was to compare the robust algorithm performance against the non-robust solution, only the classical closed-loop MMSE criterion ("MMSE") and the closed-form robust proposed algorithm ("MMSE Cond") were simulated. Clearly, all algorithms have poor performance in terms of raw-BER. Nevertheless, in terms of coded BER the new algorithm has a significant improvement compared to the non-robust solution. This behaviour is related to the robust MMSE design and to the robust Euclidean distance metrics in the soft decoding. The former guarantees a higher quality in the received symbols (see Fig 2) as the transmitted power has been more suitably allocated. The latter improves the maximum likelihood estimation since it makes better use of the CSI knowledge when considering the channel uncertainty as discussed in (12).

This paper presents a MMSE criterion for power allocation in W-LAN applications in the presence of channel uncertainties. Unlike other classical solutions, which consider an ideal CSI knowledge, channel estimation errors are taken into account in the optimizations process, obtaining an algorithm robust to channel uncertainties.

The paper derives a closed form expression for the new algorithm and shows that in terms of complexity the novel proposed technique is similar to that one of previous existing techniques. Furthermore, numerical results demonstrate that the robust solution exhibits lower sensitivity to channel estimation errors than existing algorithms.

V. CONCLUSIONS

In order to derive an expression for the cost function in (5), the statistics of \( y \) conditioned to \( x_k \) and \( \hat{\mathbf{H}} \) must be found. Let \( P[k] \) and \( E[k] \) be the DFT of the Power Delay Profile of the channel response and the channel estimation error respectively. If matrices \( \mathbf{P} \) and \( \mathbf{E} \) are built as a Hermitian Toeplitz matrices with \( P[0], \ldots, P[N-1] \) and \( E[0], \ldots, E[N-1] \) as the first row, and matrices \( \mathbf{W}, \mathbf{\hat{W}} \) are defined as:

\[
\mathbf{W} = \mathbf{P}^{-1} (\mathbf{P} + \mathbf{E})^{-1}
\]
\[
\mathbf{\hat{W}} = \mathbf{P} - \mathbf{P}^{-1} (\mathbf{P} + \mathbf{E})^{-1} \mathbf{P}
\]

then the statistics of the received data can be written as:

\[
E \left\{ \mathbf{y}_k \mid x_k, \hat{\mathbf{H}} \right\} = \mathbf{G}_k \hat{\mathbf{H}} \mathbf{F}_k \mathbf{x}_k
\]

\[
E \left\{ \mathbf{y}_k \mathbf{y}_k^H \mid x_k, \hat{\mathbf{H}} \right\} = \mathbf{\bar{w}}_{kk} (\mathbf{x}_k^H \mathbf{F}_k \mathbf{F}_k^H \mathbf{x}_k) \mathbf{G}^H + \sigma^2_k \mathbf{G}^H + \mathbf{G}_k \hat{\mathbf{H}} \mathbf{F}_k \mathbf{x}_k \left( \mathbf{G}_k \hat{\mathbf{H}} \mathbf{F}_k \mathbf{x}_k \right)^H
\]

APPENDIX I

Derivation of eqn. (8)
where \( \tilde{e}_{k,j} \) is the \( k \)-th diagonal element in \( \tilde{\mathbf{W}} \), and \( \tilde{\mathbf{H}}_{k}^{\mu} \) for the \( k \)-th carrier has been defined in terms of the coefficients in \( \mathbf{W}(w_{k,j}) \) and the channel estimates for all carriers as:

\[
\tilde{\mathbf{H}}_{k}^{\mu} = \sum_{j=0}^{N-1} w_{k,j} \tilde{\mathbf{H}}_{j}
\]

(16)

Notice that \( w_{k,j} \) measures the correlation between carriers \( k, j \), and that equation (16) is weighting all available channel estimates to reduce uncertainty in carrier \( k \). According to this notation, the mean squared error for the \( k \)-th carrier can be derived using equation (15). It can be expressed as:

\[
\tilde{\mathbf{C}}_{k}^{2} = E_{\mathbf{x}} \left\{ E_{\mathbf{y}|\mathbf{x}, \mathbf{H}} \left\{ |y_{k} - x_{k}|^{2} \right\} | \mathbf{x}, \tilde{\mathbf{H}} \right\} = \\
= \tilde{\mathbf{w}}_{k} \cdot \text{tr} \left\{ \mathbf{F}_{k} \left( \mathbf{F}_{k}^{H} \mathbf{F}_{k} \right)^{-1} \right\} \cdot \text{tr} \left\{ \mathbf{G}_{k} \mathbf{G}_{k}^{H} \right\} + \sigma_{n}^{2} \mathbf{G}_{k} \mathbf{G}_{k}^{H} + \text{tr} \left\{ \left( \mathbf{G}_{k} \tilde{\mathbf{H}}_{k}^{\mu} \mathbf{F}_{k} - \mathbf{I} \right) \left( \mathbf{G}_{k} \tilde{\mathbf{H}}_{k}^{\mu} \mathbf{F}_{k} - \mathbf{I} \right)^{H} \right\}
\]

(17)

\[
= \tilde{\mathbf{w}}_{k} \cdot \text{tr} \left\{ \mathbf{F}_{k} \left( \mathbf{F}_{k}^{H} \mathbf{F}_{k} \right)^{-1} \right\} \cdot \text{tr} \left\{ \mathbf{G}_{k} \mathbf{G}_{k}^{H} \right\} + \sigma_{n}^{2} \mathbf{G}_{k} \mathbf{G}_{k}^{H} + \text{tr} \left\{ \left( \mathbf{G}_{k} \tilde{\mathbf{H}}_{k}^{\mu} \mathbf{F}_{k} - \mathbf{I} \right) \left( \mathbf{G}_{k} \tilde{\mathbf{H}}_{k}^{\mu} \mathbf{F}_{k} - \mathbf{I} \right)^{H} \right\}
\]

(18)

### APPENDIX II

Derivation of the closed form solution

This appendix deduces the closed form solution for the cost function in (9). Replacing (10) in (8) and deriving (9) with respect to \( \Phi_{k}^{\mu} \), \( \Gamma_{k}^{\mu} \) the following equations are obtained:

\[
\begin{align*}
(\tilde{\mathbf{W}}_{k} tr \left\{ \Phi_{k} \Phi_{k}^{H} \Sigma_{k}^{-1} \right\} + \mu) \mathbf{F}_{k} + \Gamma_{k} \Phi_{k}^{H} = \mathbf{I}^{H} \\
(\tilde{\mathbf{W}}_{k} tr \left\{ \Phi_{k} \Phi_{k}^{H} \Sigma_{k}^{-1} \right\} + \mu) \mathbf{F}_{k} + \Gamma_{k} \Phi_{k}^{H} = \Phi_{k}^{H} \\
\end{align*}
\]

(19)

The solution to these two sets of equations verifies:

\[
\Phi_{k} = \frac{1}{\mu} \Gamma_{k} \Sigma_{k}^{-1} \quad \mu = C \sigma_{n}^{2}
\]

(20)

Thus, the two sets of equations collapse to one:

\[
(\tilde{\mathbf{W}}_{k} tr \left\{ \Phi_{k} \Phi_{k}^{H} \Sigma_{k}^{-1} \right\} + \sigma_{n}^{2}) \mathbf{I} + \Sigma_{k} \Phi_{k} \Phi_{k}^{H} = C \Sigma_{k}
\]

(21)

First (11) is solved for every subcarrier, if the diagonal terms of \( \Phi_{k} \Phi_{k}^{H} \) are stored in vector \( \phi_{k} \) the equations can be written as:

\[
\mathbf{A} = \tilde{\mathbf{W}}_{k} \left[ \mathbf{1}_{M \times 1} \mathbf{0}_{M \times 1} \right] + \left[ \Sigma_{k} \mathbf{0}_{M \times 1} \right] - \left[ \mathbf{0}_{M \times M} \text{diag} (\Sigma_{k}) \right] = \mathbf{A} \left[ \frac{\phi_{k}}{C} \right]
\]

\[
= -\sigma_{n}^{2} \mathbf{A}^{H} \mathbf{1} + D_{k} \mathbf{v}
\]

(22)

where \( \mathbf{v} \) is a vector spanning the null-space of \( \mathbf{A} \) and \( D_{k} \) is an arbitrary constant. Afterwards, the values for \( D_{k} \) are found by solving a second set of equations including the power constraint:

\[
\sum_{k=0}^{N-1} \Gamma_{k} \phi_{k} = P_{0}
\]

Notice that \( \phi_{k} \geq 0 \) by definition. If the obtained solution were negative for any of the subcarriers, then \( \phi_{k} \) should be set to zero, and the described procedure should be repeated for the other subcarriers.

**VI. REFERENCES**


